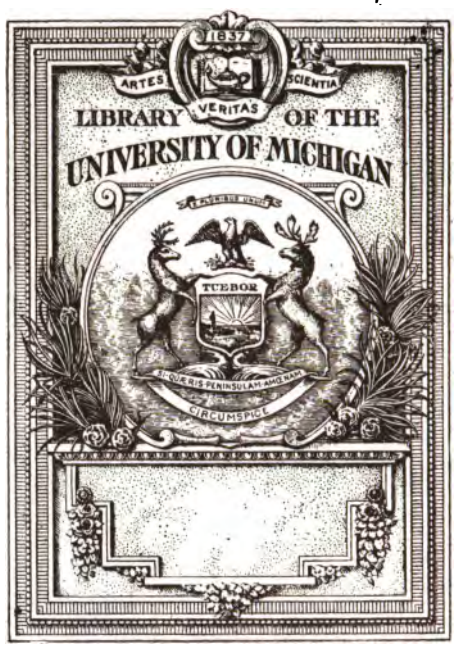
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ACTUARIAL STUDIES NO. 1

SOURCES AND CHARACTERISTICS OF THE PRINCIPAL MORTALITY TABLES

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GENERAL INTRODUCTION.

In view of the fact that, with the exception of a very few modern text books, the literature of Actuarial Science is contained in scattered original papers, The Actuarial Society of America proposes to issue a series of small volumes upon important actuarial subjects. Each volume is intended to bring together, as far as space permits, the more important points of information on the subject discussed. The objects in issuing the series are twofold: (1) to assist students of Actuarial Science, and (2) to furnish a means of ready reference for Actuaries. The various subjects are allocated to Fellows of the Society by the Committee in Charge; and, associated with the principal contributor, who is primarily responsible for the matter included and the views expressed, are one or more "Associate Contributors." These are appointed for the purpose of aiding and criticizing the work before publication. It is proposed to avoid discussing subjects already covered in the Text Book of the Institute of Actuaries except as continuity of thought may make occasional references necessary. The title chosen to represent the character of this series is "Actuarial Studies."

The thanks of the Society and of the Committee in Charge are due to all the contributors who have freely given of their time and labor, with the sole purpose of helping others—especially students.

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SOURCES AND CHARACTERISTICS

OF THE PRINCIPAL

MORTALITY TABLES

The mortality table is the scientific basis of all life insurance contracts. By means of it the probabilities of living and of dying can be ascertained, on the presumption that what has taken place in the past will be approximately reproduced in the future. A mortality table consists of a schedule showing for each age the number of persons who die and the number who survive out of a known number under observation. Usually the table starts with an arbitrary number, such as 100,000 at the youngest available age, this arbitrary number being called the *radix*, and gives for each year of age the deaths and the survivors from this original number. By adopting a radix the varying numbers which may have been under observation from year to year are reduced to a common basis suitable for comparisons, and convenient for computations of annuity values, premiums, and other functions.

The older mortality tables were formed from population statistics, but for nearly seventy years the tables which have been most valuable for insurance purposes have been derived from the records of life insurance companies. The information available to companies is usually accurate as well as complete in the numbers under observation at each age and the numbers dying, so that the rates of mortality represented by the ratio $\frac{d_x}{l_x}$ can be correctly stated for insured lives. The accuracy of this ratio is the fundamental requirement of a good mortality table; and it is of more importance that the deaths be stated at the proper ages and times than that the record of the number living should be so strictly correct, since any error in the numerator of the fraction has much more effect than a similar error in the denominator—at the younger ages the effect is more than one hundred times greater.

In forming a mortality table from population statistics, it is necessary, in order to get trustworthy figures, to have a census of the people showing the numbers living at each age, as well as the mortuary registers giving the numbers who have died at each age. The mere death rate, without distinguishing the age distribution of the people, is of practically no scientific value; the ages at death of those dying and also the ages of all who are included in the observations are essential factors. In order to ensure greater accuracy, a census at the beginning and a second census at the end of the period over which the observations extend should be taken; but fairly good approximate results can be obtained from one census only, if it be carefully made, and if the ages and deaths be accurately recorded. The mortality of nations and of cities fluctuates from year to year, and accordingly the observations should extend over a sufficient time to yield good average results.

Early Mortality Tables.

The earliest mortality tables of which we read were those in use by the Romans for determining the values of life estates. Under the Roman law a man could not bequeath more than three-fourths of his property away from the direct heir; and it was a common practice to bequeath annuities or life interests, the values of which had to be determined by computation. As the Romans made a careful census and recorded deaths with great accuracy, it is usually supposed that they had satisfactory mortality tables, but no proof of this supposition has been forthcoming. The taking of a census has now become a regular custom with civilized nations, having indeed become absolutely necessary for many purposes, such for example as the rearrangement of the methods of representation under all systems of popular government. In ancient days, however, such an act was regarded with superstition, as for example when the census of the Jews was taken by the Army of David. He believed that the pestilence from which the Jews afterwards suffered was a direct punishment for this act.

No records have been disclosed, from the days of the Romans until the close of the seventeenth century, of any reliable tables of mortality. In 1693 Dr. Halley (of comet fame), the British Astronomer Royal, published the first tables of any importance, which are now known.* He formed them from the registers of

* See J. I. A., Vol. I, pp. 43–46.

deaths in the city of Breslau in Silesia, taken for a period of five years, and they were published in the Transactions of the Royal Society (Great Britain). Before the middle of the eighteenth century a table of mortality was formed from observations made in the city of London, and from this table the original premiums adopted by the old Equitable Society were computed. The mortality rates were very high.

In 1746 M. De Parcieux published his "Essai sur les probabilités de la Durée de la Vie Humaine" in which several tables of mortality were given, constructed from the lists of nominees in the French tontines and from the mortality registers of different religious houses. The mortality shown by this table is generally higher than that of the Carlisle Table, fully described hereafter, with which in other respects it agrees quite closely. This table was never freely used by English-speaking people, although it was more accurate than any of the other tables published in the eighteenth century. It has now only a historical interest.

In the early development of scientific mortality tables we owe more to Dr. Richard Price than to any other individual; yet he is popularly remembered as the author of the Northampton Table which is held out as an example of faulty construction. Dr. Price himself was well aware of the difficulties with which he had to contend and he strongly urged the adoption of better systems of registration so that accurate mortality tables could be formed. Amongst other tables formed by Dr. Price were the CHESTER TABLES,* which gave the rates of mortality separately for males and females, and for the time at which they were prepared gave a fair conception of human life. The tables were formed from the birth and burial registers of the small town of Chester, in England, and the accuracy of the assumptions was tested by an enumeration of the people which included a record of the numbers living at each age. Unfortunately the tables never came into general use, and the Northampton Tables became much better known.

* See the later editions of "Price on Annuities."

Northampton Table.

The first tables used to any great extent for life insurance purposes were the Northampton Tables, detailed information regarding which is here given as the mode of construction calls attention to one or two errors which should be avoided. Moreover they are still prescribed under certain old statutes as the basis for determining the values of life estates for taxation purposes.

There were two tables formed by Dr. Price, the first published in 1771 in his "Observations on Reversionary Payments," and the second, a more complete table, in 1783, the latter being known as *The* Northampton Table. The following extract from Dr. Price's remarks will partially explain the Tables:

"In the parish of All Saints, containing the greatest part of the town of Northampton, an account has been kept since 1735 of the ages at which all have died, also an account of the number of males and females that have been christened.

	Christened 4220	Buried 4689
"Of these died under	2 yrs. of age	1529
aged	2 to 5	362
	5 to 10	201
	10 to 20	189
	20 to 30	373 (351)
	30 to 40	329 (351)
	40 to 50	365
	50 to 60	384
	60 to 70	378
	70 to 80	358
	80 to 90	199
	90 to 100	22
		4689

"In the fourth edition of this treatise, the following corrections were made:

"1st. The table printed in the first three editions having been formed from the Northampton Bills (of Mortality) for 36 years (i. e., the first Northampton Table 1735–1770) this table was rendered a little more correct in consequence of being formed from the same Bills for 46 years (1735–1780).

"2nd. The Bills give the number dying between 20 and 30 greater than between 30 and 40, but this being a circumstance which does not exist in any other register of mortality, and undoubtedly owing to some accidental and local causes, the decrements were made equal between 20 and 40, preserving the total of deaths the same that the Bills have them.

"3rd. The Bills giving only the total of deaths under two years of age and between 2 and 5, the proportion of deaths for every particular year between 2 and 5, and for every quarter of a year after birth till one year of age, were made the same nearly that the Chester register makes them."

Dr. Price formed the table by taking account only of the deaths and without using any enumeration of the population. If a census

had been used a different result would have been reached, and the tables would have been much more accurate. In view of the difference between the number of christenings and of burials (469 or 10% of the burials) he assumed that not only 10% but 13% of the deaths were those of persons who immigrated to Northampton at the age of twenty; his reason for adopting the higher percentage is not stated. The excess of deaths over births was probably caused almost entirely by deaths among Dissenters, whose children were not entered in the Parish Register of Christenings. Dr. Farr pointed out that if effect had been given to the increasing nature of the population, the table would have shown lower and more accurate mortality rates, and would have been much more valuable. The mortality amongst male children appears heavier than amongst females, a feature which has been confirmed in other tables.

CONSTRUCTION OF THE NORTHAMPTON TABLE.*

Age.	Deaths.	Numbers Living at First Age in Group (if Population had been Stationary).	Numbers in Column 3 Raised to Radix of 10000.	Column 4 Less 1300 up to Age 20 (i. e. 13 % of Radix).	Column 5 Raised in the Proportion of $\frac{11649}{8700}$ up to Age 20.
(1)	(2)	(3)	(4)	(5)	(6)
0– 2	1529	4689	10000	8700]	11649
2– 5	362	3160	6739	5439	7283
5–10	201	2798	5967	4667	6249
10–20	189	2597	5538	4238	5675
20–30	373 (351)	2408	5135	3835	5135
30–40	329 (351)	2057	4387		4387
40–50	365	1706	3638		3638
50–60	384	1341	2860		2860
60–70	378	957	2041		2041
70–80	358	579	1235		1235
80–90	199	221	471		471
90–100	22	22	47		47
	4689				

The figures given by Dr. Price differ very slightly from those given in the last column, and are probably the result of an arbitrary adjustment.

The mortality shown by the Northampton Table was excessively high, and this led to large profits being made by the early insurance companies, in which a much lighter rate of mortality was experi-

* See J. I. A., Vol. XVIII, p. 107 et seq.

enced than the table led them to expect. On the other hand, annuity companies and the government through their annuities, lost heavily. Dr. Price was himself sensible of the very high mortality rates shown by the Northampton Table, and in 1782 he recommended the Equitable Society to adopt the lower rates given in the Chester Experience; but an excess of caution on the part of the society gave the Northampton Table a standing which it never deserved.

The mortality shown by this table has been found to correspond roughly to the mortality of certain colored races; and, as very complete tables of monetary values are available on the Northampton basis, the tables may in rare instances still be used with advantage, when investigation shows that some class of substandard mortality approximates to that of the old table.

As far back as 1823 the faults of these tables were well known, as the following quotation shows:

"Dr. Price did as much as the nature of his materials would allow. For in those days no census or enumeration of the population had been made; and without a comparison of a census (in which the ages are carefully distinguished) with the Bills of Mortality, an accurate Table of Observations cannot possibly be obtained." FARREN on *Life Assurance*, 1823

THE CARLISLE TABLE OF MORTALITY.*

(*Published 1815.*)

This table was formed by Joshua Milne, Actuary to the Sun Life Office, London, from materials contained in a tract published by Dr. John Heysham at Carlisle in 1797. These materials consisted of a careful census in January, 1780, and a less accurate enumeration in December, 1787, of the two parishes in Carlisle, St. Mary and St. Cuthbert, together with the Register of the deaths in those parishes during the nine years, 1779 to 1787.

The original statistics were divided into male and female; and the deaths showed also the condition of each sex as regards marriage; but the graduated tables dealt only with the total of both sexes, details of which are given in the table quoted on the next page.

It has been pointed out that in the second enumeration the ages do not appear to have been taken, but that the increment 1000 seems to have been proportionately added to the first enumeration,

* Milne on Annuities and Assurances, London, 1815, Article 704, p. 404 et seq. See also King, J. I. A., XXIV, 186.

Between the Ages.	Population		Deaths in the 9 years 1779–1787.	Between the Ages.	Population		Deaths in the 9 years 1779–1787.
	in Jan., 1780.	in Dec., 1787.			in Jan., 1780.	in Dec., 1787.	
0– 1			390		3327	3761	979
1– 2			173	20–30	1328	1501	96
2– 3			128	30–40	877	991	89
3– 4			70	40–50	858	970	118
4– 5			51	50–60	588	665	103
				60–70	438	494	173
0– 5	1029	1164	812	70–80	191	216	152
5–10	908	1026	89	80–90	58	66	98
10–15	715	808	34	90–100	10	11	28
15–20	675	763	44	100–105	2	2	4
	3327	3761	979		7677	8677	1840

i. e., in the ratio $1000 \div 7677$. This increment looks like an approximation to the true population and Dr. Heysham said of it, after correcting some errors, "I am persuaded the enumeration is *now pretty exact.*"

Adding the totals together (7677 + 8677) the result was multiplied by four in order to place the population and the deaths on a common basis for comparison, the latter being reduced by one-ninth in order to effect this. The result was an approximation to the sum of the total population exposed during eight years, and this was compared with an approximation to the sum of the deaths during those eight years.

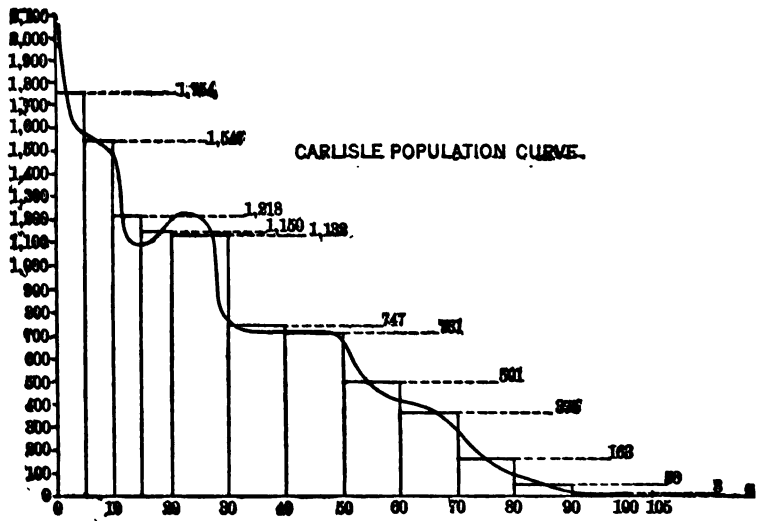
The following table shows the results thus obtained:*

Between the Ages.	Population.	Deaths.	Between the Ages.	Population.	Deaths.
0– 1		347.0		28352	870.5
1– 2		153.7	20–30	11316	85.3
2– 3		113.9	30–40	7472	79.1
3– 4		62.2	40–50	7312	104.9
4– 5		45.3	50–60	5012	91.6
			60–70	3728	153.8
0– 5	8772	722.1	70–80	1628	135.1
5–10	7736	79.1	80–90	496	87.1
10–15	6092	30.2	90–100	84	24.9
15–20	5752	39.1	100–	16	3.6
	28352	870.5		65416	1635.9

The results given in the above table were graduated by a graphic method of adjustment, two graphs being prepared, one for

* Sutton, J. I. A., Vol. XXIV, pp. 110–122.

the population and one for the deaths. The following is a geometrical illustration of the population graph:



The base line represents the age attained while the upright lines (called ordinates) represent the numbers alive at the various ages. The areas of the parallelograms represent the total population in the several groups; thus, the base of the first parallelogram being 5 years, and its area 8772, its altitude is $8772 \div 5 = 1754$. Similarly, the altitudes of the next 3 parallelograms are $7736 \div 5$, $6092 \div 5$, and $5752 \div 5$ respectively, but the base of the fifth being 10, its altitude is $11316 \div 10$, and so on for the other altitudes.

A curved line was then drawn through the parallelograms proceeding smoothly from the one to the other, adding a portion to each, and cutting off an equal part. The areas were then subdivided into fifths or tenths as the case required, each subdivision representing a period of one year. These years were bisected, and the length of the ordinate then erected gave the graduated population at the middle of the year, i. e., L_x, which Mr. Milne wrote $\overset{s}{L}$. The deaths were treated in exactly the same way, and $\overset{s}{D}$, i. e., the mean number that died between the ages x and $x + 1$, was obtained. From these mean values Mr. Milne formed the values of d_x. The radix was taken as 10,000 and the figures after age 100 were adjusted arbitrarily.

Females represented about 55% of the population, and this had the effect of showing light mortality at the older ages, one of the characteristics of female mortality being that it is much lighter than male after the age of 50. Amongst insured lives, only about 10% are females, so that the table was never a good one for life insurance premiums, or valuations; and it is not now used for such purposes. The original statistics and remarks published by Mr. Milne in 1815 contain a great deal of valuable information, including comparative mortality—(1) from different diseases, (2) during different seasons of the year, and (3) of the two sexes. The original publication is well worthy of careful perusal even at this date.

The results of the graduation left several irregularities, especially at the older ages. For example, at age 91, the expectation of life is 3.26 years, while at age 95, when it should be less, it is 3.83. This was caused by the system of graduating separately the population and the deaths, whereas a regraduation should have been made of the ratio m_x. The irregularities were completely removed in a graduation which was successfully accomplished by Messrs. King & Hardy* in 1880 by applying Makeham's Formula. This formula gives a perfectly smooth curve when the results are presented graphically and the general characteristics of the original Carlisle Table were maintained from childhood till old age. It is only when approaching age 90 that the difference between the two tables becomes considerable. Facilities were thus given for the calculation of the more intricate benefits.

The table has been very extensively used, and even yet is of considerable value in calculating the values of Reversions, etc. As the published tables based on Carlisle Mortality are unusually extensive, it is often adopted as a matter of convenience in connection with special calculations.

English Life Table No. I.†

The compulsory registration of births, marriages, and deaths was introduced in England in 1836, and this requirement of law, which became operative from July, 1837, gave a proper basis for the formation of a national table of mortality.

Dr. William Farr was appointed Compiler of Abstracts in the

* J. I. A., Vol. XXII, p. 221.

† Registrar General's Returns as quoted. Walford's Encyclopedia. Farr's "Vital Statistics, 1885."

General Register Office in 1839 and for the next forty years he was engaged in developing a national system of vital statistics. He had no official connection with the census enumeration of 1841, the observations being then made on a plan recommended by Professor de Morgan and Mr. Griffith Davies. But Dr. Farr used the census enumeration in conjunction with the Registers of deaths to compile the first English Life Table which was published in the Registrar General's fifth annual report, 1843. This table was based upon the Census Returns for England and Wales made on June 6, 1841, and the deaths taken from the Registers of that year. As the deaths related to the calendar year, it was necessary to make a correction in the population for the period from June 6 to July 1—the middle of the year. An increase of 1.334% had taken place in the ten years from 1831 and it was assumed that this rate of increase continued during the 25 days, or .07 of a year, until July 1st. The population as adjusted to July 1st was 15,927,867 and the deaths 343,847. In the census enumeration some of the ages were not specified; and in such cases they were assumed to be in the same proportion as those whose ages were given. In like manner the ages at death in 521 cases were not given and were similarly treated.

The rates of mortality in a single year cannot be viewed as a proper basis for a national table, because even with a large population of nearly 16,000,000 the variations caused by climatic conditions, epidemics, etc., are great. It appears, however, from subsequent results that the year 1841 did prove to be a fair average one.

Owing to the tendency for persons at any age between 30 and 40 to state their age as 30, Dr. Farr said that in quinquennial groups the death rate for the period from age 30 to age 35 would be too small and for the period from 35 to 40 too large. He further said: "It is my opinion that the ages of the people have been returned with sufficient accuracy for all practical uses in *decennial periods.*"

To overcome such difficulties and obtain rates of mortality for each age, Dr. Farr dealt with the statistics as if they were two series of geometric progressions—the first from age 15 to age 55 and the second from age 55 to age 95. He obtained the ratios for these geometric series by comparing the increase in the mortality in the groups—

First	15–20		25–30		35–40		etc.			
Second		20–25		30–35		40–45		etc.		

Since the errors in these two series would be of a similar nature there was a great probability that the ratio would be satisfactory. From such quinquennial grouping a central death rate was obtained and treated as the death rate applicable to the central age of the group. The table was based upon 100,000 births of whom 51,274 were males and 48,726 females—being the proportion in which the births of the two years 1840 and 1841 were distributed. The table therefore represents a generation of 100,000 individuals born at the same instant and shows the relative number of males and females at each age, as well as their probabilities of living and expectations of life.

The period of infancy was differently treated and Dr. Farr explained the method as follows:

"The births of boys in the two years 1840–41 amounted to 520,157; which was at the rate of 260,078 a year; whence it may be assumed that 260,078 were born in the year, of which 1 January, 1841, was the middle—or that the mean date of their birth was January 1, 1841. We can then reason upon the assumption that 260,078 boys—the mean ann. number of boys born in 1840 and in 1841—were born 1 January, 1841. But all the boys who *died in* 1841, under 1 year of age, must have been boys born in 1840 and in 1841. The deaths occurred in the year 1841—in one year—and they must therefore be compared with the births in one year, viz., with the births in the year of which January 1, 1841, was the middle. We have then this result, that 41,444 of 260,078 boys born died in the first year after birth. . . .

"All the births are not registered: the deaths in the first year must have occurred out of more than the number of births returned; and the mortality in the first year must have been less than that given in the table, which is, however, lower than the mortality deduced immediately from the children stated at the enumeration to be living at the first year of age, and the deaths registered at the same age. As it is, the mortality in the first year stands lower than in any other authentic table."

As large masses of figures were involved in each age-group a graduation of the group results was neither necessary nor desirable. To get results at individual ages, "the mortality at every age was interpolated by the log which expresses the ratio of increase in the mortality at every year of life, and the chance of living each year was deduced from $\frac{1 - \frac{1}{2}m}{1 + \frac{1}{2}m}$."

English Life Table No. II.

This table was published in the 12th Report of the Registrar General, 1853, the census of 1841 again furnishing the basis for the exposures, but the deaths were taken for a period of seven years from January 1, 1838, to December 31, 1844, inclusive, the total deaths in this period being 2,436,648. During that time there were no epidemics of any consequence. Dr. Farr calls attention to the rapid variation in the rates of mortality in infancy, "when the rate of mortality varied so rapidly that every year and even month is marked by a change." At the earlier ages the mortality rates were directly taken from the return but after age five, quinquennial, and after age fifteen decennial periods were used. The table is of little importance in comparison with the later compilation with which the results are in general agreement. The final tables were based upon a radix of 10,000,000 persons—5,126,235 males and 4,873,765 females.

English Life Table No. III.*

Published in 1864 as a distinct work by Dr. Farr. Several improvements were introduced in construction and the Table is worthy of description in greater detail than either No. I or No. II.

Two censuses were used—those of 1841 and 1851, comprising 15,929,492 persons as adjusted to the middle of 1841 and 17,982,-849 in 1851, while the deaths extended over the 17 years from 1838 to 1854—that is 3½ years approximately before the first census and a similar period after the second census. During that time there were 6,470,720 deaths.

The population was carried back to the beginning of 1838 by using the ratio of increase in the population from 1841 to 1851, assuming that such increase is in geometrical progression. Thus, having obtained the ratio from the formula

$$\log r = \frac{\log P_{51} - \log P_{41}}{10},$$

where P_{51} represents the population in 1851, the population at the

* "English Life Table," Longman, Green & Co., London, 1864.
Walford's Insurance Cyclopaedia. Farr's "Vital Statistics," 1885.
Important. J. I. A., Vol. XLII, p. 228 et seq. (King).

beginning of 1838 was estimated by the formula

$$\log P_{38} = \log P_{41} - 3\tfrac{1}{2} \log r.$$

The following table shows the population of England and Wales estimated in the middle of 1841 and 1851, in Dr. Farr's groupings.

Ages.	Persons.		Males.		Females.	
	1841	1851	1841	1851	1841	1851
All ages.	15,929,492	17,982,849	7,784,883	8,808,662	8,144,609	9,174,187
0–	2,107,008	2,355,345	1,048,270	1,180,430	1,058,738	1,174,915
5–	1,865,856	2,098,808	953,235	1,053,510	912,621	1,045,298
10–	1,772,913	1,919,255	880,567	967,007	892,346	952,248
15–	3,145,541	3,418,488	1,511,602	1,671,634	1,633,939	1,746,854
25–	2,450,322	2,740,919	1,174,473	1,323,621	1,275,849	1,417,298
35–	1,778,737	2,089,629	875,874	1,017,018	902,863	1,072,611
45–	1,270,178	1,516,324	617,113	734,314	653,065	782,010
55–	832,692	1,010,973	399,490	482,788	433,202	528,185
65–	483,593	579,187	224,310	268,052	259,283	311,135
75–	190,443	220,618	86,736	97,008	103,707	123,610
85–	30,541	31,754	12,635	12,745	17,906	19,009
95 and upwards	1,668	1,549	578	535	1,090	1,014

By arranging the groups in this way a correction is applied to the tendency in census returns to state the ages at the nearest decennial, as 30, 40, etc. The deaths for the same years were arranged to correspond and thus average annual rates of mortality were obtained by dividing the deaths during the 17 years by $8\frac{1}{2}$ times the sum of the populations in 1841 and 1851. This was apparently a departure from the principle of geometric increase; but it was found by actual trial that the geometric mean (1838–1854) was almost identical with the method adopted.

Having thus obtained the Central Death Rate for the middle age of each group Dr. Farr calculated the probabilities of living by a logarithmic method which gave results he said nearly identical with the formula

$$p_x = \frac{2 - m_x}{2 + m_x}.$$

Values of p_x for intermediate ages were then interpolated by finite differences; this method of construction made graduation unnecessary.

As in Table No. I the statistics were scheduled in three parts:

1. Persons—Male and Female.
2. Males.
3. Females.

The radix of the first part was 1,000,000 children born alive; of the second part 511,745, and of the third part, 488,255, being the proportions in which boys and girls were born during the seventeen years under observation. The male and female tables were constructed independently; that of the "persons" was obtained by combining the other two in one.

Out of 1,000,000 persons born, it was shown that 149,493 died in the first year; 53,680 in the second; with decreasing numbers of deaths until the age of 13, when 3,382 died between that age and 14; then this number increased steadily for each age until it reached 15,469 who died between 73 and 74. Thereafter, although the rates of mortality increase rapidly the annual deaths diminish; 92 died at the age of 100 and the last one at the age of 108—"so 109 years is the limit of age of this table."

So far as such census tables can be computed from data which are admittedly imperfect, being based on voluntary unverified statements, the table is looked upon as a great advance on older methods, and many of the ideas used for the first time in that table have since been copied and adopted by other statisticians. Dr. Farr in all three tables assumed that the average value of the death rate for an age interval gave the true value for the central point of age.

Census tables represent the mortality of all classes mixed. This means a preponderance of the laboring and industrial classes; but the table cannot be said to represent even these classes, because of the percentage of well-to-do people. For financial calculations applicable to individuals the tables do not meet with present-day requirements.

Mortality in Infancy.—From birth until the age of 1 the deaths were given for each month of age. They were extracted from the registers of births and deaths directly, the census figures being of no value for this particular purpose. For the 17 years under observation the deaths under age 1 were given for (*a*) the first 3 months, (*b*) the second 3 months, and (*c*) the next 6 months; but for 8 years of the period Dr. Farr abstracted the deaths for each one of the

first three months, and then distributed in the same proportions the deaths during the 17 years occurring in the first three months. The other figures for the first twelve months were obtained by interpolation. These deaths in conjunction with the total number born admitted of a good approximation to the probabilities of living and dying.

The English Life Table No. III shows slightly higher mortality rates than either No. I or II, as indicated by the Expectations of Life at age 30 (male). The cause of this slight difference lies probably in the processes employed, rather than in reduced vitality.

English Life No. I = 33.13 years.
No. II = 33.21 "
No. III = 32.76 "

The volume published in 1864 not only explained the construction of the experience but gave also about 600 pages of tables with monetary values and commutation columns from 3% to 10%, also complete joint-life values at 3% for (1) two males, (2) two females, and (3) male and female. The tables have been freely used by Industrial Insurance Companies.

Healthy English Life Table.*

After the completion of the English Life Table No. III Dr. Farr set himself to the construction of another table based upon the experience of 63 of the healthiest English districts inhabited by nearly a million people, dividing the experience into males and females. He gave a brief description in his "Vital Statistics" as follows:

"The Healthy District Life Table was constructed in 1859 from the Census enumeration of 1851 and from mortality observations extending over the five years 1849 to 1854 in 63 districts of England and Wales which showed during the ten years 1841–50 a mean annual death-rate not exceeding 17 per 1,000 persons living. It has been found by experience that this Healthy District Life Table expresses very accurately the actual duration of life among the clergy and other classes of the community living under favourable circumstances."

In general, the methods of construction were those used in compiling the English Life Table No. III, with slight modifications.

* J. I. A., Vol. IX, pp. 124, 188, etc. Farr's "Vital Statistics." Transactions of the Royal Society, 1859, pp. 838–41.

The probabilities of life for the first four years, namely p_0, p_1, p_2, and p_3, were also obtained from the births and deaths without reference to the census figures. For example, half the number of births in two calendar years was taken as the number exposed to risk from January 1 of the second year, while the death records gave the numbers dying under age 1, from 1 to 2, from 2 to 3, etc., in the subsequent years, whence it was easy to form l_x and d_x columns, and get the early values of p_x. Three such computations were independently made by using different years, and the values finally adopted were the mean of the three. Values of p_7, p_{12} and p_{20} were obtained from the census figures, and the intervening figures were computed by mathematical interpolation from p_3 to p_{20}. Dr. Farr pointed out that if there were much emigration from a community this would result in an understatement of the death rates, because the births would be properly recorded, but children would be taken away by their parents and would die elsewhere. This effect would be heightened each year after birth, and at ages 4 and 5 the death rates might be appreciably affected. On the other hand a failure to register births might have the opposite effect.

Later English Life Tables.

The fourth* English Life Table was based on the mortality for the decennium 1871 to 1880 and founded upon the preceding table (No. 3) by making allowance for the different rates of mortality in the new period. Details regarding it are of little interest.

The fifth* and sixth* English Life Tables were prepared by Dr. John Tatham and the improvements in construction, as well as the increased healthiness at the younger ages to which they direct attention, are so considerable as to demand adequate mention notwithstanding the fact that Dr. Tatham himself said in 1907 "The scheme devised by Dr. Farr in 1864 has been in all essential points adhered to by his successor." The fifth Table was based upon the mortality in the ten years—1881 to 1890—and the methods of construction were the same in principle as the sixth Table, differing only in minor matters, such as the groupings from

* These Tables are all published in the supplements to the annual reports of the Registrar General of Births, Deaths and Marriages in England and Wales and appear in the 45th, 55th, and the 65th, (1907) annual reports respectively.

ages 5 to 25, and the functions used for interpolation. The new methods are free from the serious error of Dr. Farr's assumption mentioned on p. 14, and Mr. George King says, "The principles underlying them can scarcely be improved upon."

The facts collected by the census enumeration and those taken from the death registers were grouped round decennial points from ages 20 to 90. Thus, the decennial point 30 was the center of the grouped facts from age 25 to age 35. Each group therefore represented the population living between ages x and $x + n$ and the deaths between the same ages. In actuarial notation these groups can be expressed as $T_x - T_{x+n}$, or $T_{x\overline{n}|}$, for the population and $l_x - l_{x+n}$, or $l_{x\overline{n}|}$, for the deaths.

By summing all the groups from x to the oldest age we obtain the values of T_x and l_x for certain ages, 25, 35, 45, etc., and from these we can deduce successively the intermediate values of T_x and l_x, as well as complete values of L_x, d_x, m_x, p_x, and other functions as desired. Mr. King* recommends that T_x and l_x be interpolated separately by a formula of osculatory interpolation to get the intermediate values. But the method actually employed in the English Life Table No. 5 was to interpolate by ordinary differences between the successive decennial values of log $(2T_x + l_x)$ and log $(2T_x - l_x)$ so as to get log $(2T_{x+1}+l_{x+1})$ and log $(2T_{x+1}-l_{x+1})$ whence by differencing the natural numbers, decennial values of $(2L_x + d_x)$ and $(2L_x - d_x)$ were found. From these functions the decennial values of p_x were obtained from ages 25 to 65. Other methods were employed at younger and older ages. Then the logs of decennial values of p_x from age 5 to age 85 were interpolated to obtain the figures for each age.

Overlapping values were used in the interpolation, and two sets of probabilities were obtained for each age. A more regular curve can be obtained by using an overlapping series rather than an abutting series. From the values at ages 25, 35 and 45 one set of probabilities for 35 to 45 was obtained, and from ages 35, 45, and 55 another set. The junction between these two partial curves is generally smoother than the junction between two partial curves 25, 35, 45, and 45, 55, 65—the former being overlapping and the latter abutting. Each probability in the first series was then multiplied by an empirical fraction so as to give greater weight to the terms nearest the central decennial value. Nine

* J. I. A. XLII, p. 236.

factors were derived from the "Curve of Sines" and ranged from .02447 to .97553, the central value being .50000. The same probability in the second series was multiplied by the complementary fraction, and the addition of the two gave the final value. For example, p_{36} would be derived from two sets of interpolations, the first set having its central decennial point at age 35, and the second set its central decennial point at age 45; accordingly the first value of p_{36} would be multiplied by .97553 and the second by .02447—the sum of the two giving the final result.

For English Life Table No. 6 based on the mortality of the ten years 1891–1900, the functions interpolated were $\log 2T_x$ and $\log l_x$, these functions being developed apparently with the view of using the formula

$$p_x = \frac{2L_x - d_x}{2L_x + d_x};$$

but there is no apparent advantage in either the No. 5 or No. 6 plan, Mr. King's suggestion giving more direct results with greater ease.

The values above age 85 were obtained by extrapolation from the five values 45, 55, 65, 75, and 85, because census returns at the older ages are unreliable. The system of extrapolation cannot be considered as satisfactory, but it was a case of choosing the better of two doubtful methods.

Probably the most important innovation in the Mortality Statistics of the 5th and 6th English Life Tables deals with the *Mortality by Occupation.*—The varying rates of mortality in different occupations first formed the subject of an inquiry by Dr. Farr about 50 years ago, and his ideas have been developed and improved upon until a very complete analysis of this nature has now been given for the triennial period 1900 to 1902, based upon the census figures of 1901. The numbers living in 1901 were obtained from the census, and the deaths in each occupation from the registers. The information which relates only to male lives was then divided into four principal areas:

1. London,
2. Industrial districts,
3. Agricultural districts, and
4. Other parts.

Comparisons of this nature are generally made in other countries from "crude" death rates, which show the number of deaths per 1,000 of population without giving effect to age or sex distribution, and are thus often misleading; but the recent publications of English Mortality give "corrected" figures. A standard table is adopted showing the numbers alive in decennial periods in an average population, and a corrected death rate for the particular occupation or locality is then deduced by reducing the population to the standard basis for comparison. The system is not perfect, but it shows a great advance towards scientific accuracy, and the necessity for such treatment is well illustrated in the case of farmers. The crude death rate amongst farmers, ages 15 and upwards, for 1900–2 was 16.44 per 1,000, whereas the general death rate amongst all males of 15 and upwards was only 16.23. From this it would appear as if farmers were subject to a higher death rate than males generally; yet at each age group the death rate amongst farmers is much *lower* than the death rate of males generally, and the high crude death rate is caused by the large number of farmers at the older ages, and, relatively, the small number at the younger ages.

The principal comparisons were made during the active working period of life, in four decennial age groups from 25 to 65. Over 100 occupations were dealt with—in some instances industries of like character being grouped under the same heading. The census tables of 1901 also supply information about those who had retired from any particular calling—the word "retired" covering those who had retired in comfort as well as those whose health had broken down and had thus lost the ability to earn a living. The occupation dealt with is that which has last been followed, and no system has yet been devised for dealing with changes.

In the period from ages 25 to 65 there had been a consistent decline in mortality between 1890–2, and 1900–2, this decline varying from 11 per cent. at ages 55 to 65 to 17 per cent. at ages 25 to 35. Graphic illustrations are given showing the gradations of the mortality for the age group 25–65 in different occupations commencing with clergymen, gardeners, game keepers, and farmers, who show the lowest corrected death rates, and ending with hotel servants, costermongers, tin miners, and general laborers, who show the highest death rates.

In some instances the figures must be used with great caution

because the classified numbers become so small that accurate deductions from them are not warranted. Moreover, some occupations requiring muscular strength and vigor are forsaken in favor of an occupation of lighter character when a man falls into ill health in the former. These occupations, therefore, exercise a species of selection favorable or adverse by attracting to themselves either the unusually vigorous or the weak. This probably had much to do with the heavy death rates in the miscellaneous occupations like costermongers and general laborers. During the period 1890–1892 there was prevalent a serious epidemic of influenza, which made the mortality at that time abnormally high. Accordingly the lower rates shown ten years later are probably not caused solely by improvement in longevity. The census date —1st April, 1901—was not exactly in the middle of the three elementary years during which the deaths were computed, but it was thought better to neglect the small error thus introduced rather than to make estimates of the subdivided populations in various occupations to the exact middle of the year 1901, and it was found impracticable to take the deaths otherwise than for the three complete calendar years.

In the statistical tables the deaths in the various occupations are not only given in seven age groups, but are given under "causes of death," and are also published separately for those who are occupied and those who are "occupied and retired." The mortality of industrial districts is above, and of agricultural districts below, the average.

Healthy English Life Tables Nos. II and III.

The utility of a mortality table deduced from healthy districts only, as recommended and originated by Dr. Farr, has been confirmed and approved by later statisticians. Dr. Farr based his table upon those districts in England and Wales whose *crude* death rates did not exceed 17 per 1,000, one district with larger death rate having been included through error. As the death rates were tabulated to the nearest whole number, this practically meant under 17.5 per 1,000.

A second Healthy English Table was formed after the census of 1891, and at that time a "corrected" death rate was used showing the number of deaths per 1,000 in each registration district if the ages and sex distribution had been in the same proportion as the

general population at the preceding census. The second Healthy Districts Table was formed from the statistics of England and Wales over the period from 1881 to 1890, and a third Healthy Districts Table was formed for the period from 1891 to 1900.

Lower rates of mortality were recorded during each of these decennial periods than had been shown in Farr's earlier table, and in forming the second table those districts only were used where the *corrected* death rate was *15 per 1,000 or less.* There were 263 of such districts with an aggregate mean population of 4,603,-055, representing during the ten years over 46,000,000 years of life, or more than nine times as many as the older table.

In forming the *third table* the standard of healthiness was again raised and 260 districts were included whose aggregate population was 4,477,485 with *corrected death rates not exceeding 14 per 1,000.* Of the 260 selected districts 222 were common to the second as well as the third table. Accordingly 41 of the districts used for the second table were excluded in forming the third table, and 38 new districts were included. When reduced to a common standard, after making allowance for age and sex distribution, the corrected death rate of the entire group of healthy districts for the third table was .85 per 1,000 lower than that of the second table. Had the corrected death rate of 15 per 1,000 been used (in 1891–1900) there would have been 352 districts (with an aggregate mean population of 7,326,280 persons) qualifying under such standard for the third table.

The districts which experienced the low death rates were almost exclusively those which are rural in character or contain only small towns with rural surroundings. No part of London is represented in the new table, yet of the healthy districts, a large proportion is in the section of England of which London might be taken as the center. The progress of healthfulness in England and Wales may be partly indicated by Tables of Expectations as herein submitted; but it should be borne in mind that the figures are affected by the inaccuracies already discussed.

Table.—Males.	Expectations of Life.				
	Age 0.	Age 20.	Age 35.	Age 50.	Age 65.
English Life No. 3 (1838–1854)	39.91	39.48	29.40	19.54	10.82
" " " 4 (1871–1880)	41.35	39.40	28.64	18.93	10.55
" " " 5 (1881–1890)	43.66	40.27	28.91	18.82	10.31
" " " 6 (1891–1900)	44.13	41.02	29.24	18.90	10.34
Healthy Districts No. 1 (1849–1853)..	48.56	43.40	32.90	22.03	12.00
" " " 2 (1881–1890)..	51.48	44.41	32.70	21.53	11.60
" " " 3 (1891–1900)..	52.87	45.37	33.32	21.74	11.61

Table.—Females.	Expectations of Life.				
	Age 0.	Age 20.	Age 35.	Age 50.	Age 65.
English Life No. 3 (1838–1854)	41.85	40.29	30.59	20.75	11.51
" " " 4 (1871–1880)	44.62	41.66	30.90	20.68	11.42
" " " 5 (1881–1890)	47.18	42.42	31.16	20.56	11.26
" " " 6 (1891–1900)	47.77	43.44	31.52	20.64	11.27
Healthy Districts No. 1 (1849–1853)..	49.45	43.50	33.46	22.87	12.58
" " " 2 (1881–1890)..	54.04	45.62	34.16	22.75	12.36
" " " 3 (1891–1900)..	55.71	46.93	34.79	22.92	12.36

English Life Tables Nos. 7 and 8.

These two tables were published at the same time, in 1914, in the form of a Supplement to the 75th Annual Report of the Registrar General of Births, Deaths, and Marriages in England and Wales. In an Appendix all of the statistics are published which were used to form the final tables; also the mathematical formulas employed.

Table No. 7 consists of the experience during an intercensal period of ten years from 1901 to 1910 inclusive. Table No. 8 deals with the deaths in three calendar years 1910, 1911, and 1912, grouped around the census point in 1911, the population being adjusted to 1st July, 1911. Both investigations were made by Mr. George King, and his report on the work contains a lucid description of the tables and of the reasons for the decisions which were reached on points of difficulty.

Table No. 7 follows in its general principles the main lines of investigation which characterized, and developed from, the earlier English Life Tables; but a radical change in mode of construction was made in the adoption of King's method. The population for each year to age 4 inclusive was enumerated, then in quinquennial groups 5 to 9 last birthday, 10 to 14, etc. The deaths from 1901 to 1910 were given in decennial groups from

age 25 onwards; but were subdivided into quinquennial groups in the proportions existing among the deaths of 1910 to 1912. From the said quinquennial values pivotal values of population and deaths were obtained, ages 12 to 97 inclusive—hence m_x, etc. From these central death rates, quinquennial values of q_x were deduced. Then by osculatory interpolation, using the function log $(q_x + .1)$, the values from 18 to 97 were obtained.

The rates of mortality at ages 0 to 4 inclusive were determined from the death returns at individual ages and the readjusted average population. This readjustment was found necessary because the census returns indicated a deficiency of population at ages less than two last birthday.* The theoretical population for each attained age was accordingly calculated from the birth and death returns and each age was then reduced in the proportion necessary to bring the total exposed to risk under age 5 down to the total derived from the census returns. The assumption that the total population at ages under five was correctly given in the census returns was made because a similar assumption had been made in the construction of English Life Table No. 6, and it was desired that the two should be comparable. The birth and death returns, however, indicate that the deficiency at the two younger ages was an absolute one and not a transfer to older ages. The rates of mortality at ages 5 to 16 inclusive were then inserted by Lagrange's *interpolation* formula from the values for ages 3, 4, 12, 17 and 18.

At the extreme old ages no assumption was made with regard to the termination of the table. The original data were used to age 92, with a pivotal value at age 97. A fourth difference series of values of log p_x was determined from the values at ages 89, 90, 91, 92 and 97, and projected onwards. While there is no theoretical limit the practical limiting age is about 110 for men, and 114 or 115 for women. It is claimed that the new tables are more accurate than previous English Life Tables at the older ages

Important changes in methods of construction were made in forming Table No. 8. These may be briefly summarized as follows:

* The accuracy of these deductions by Mr. King has been questioned by Dr. J. C. Dunlop's able analysis, with verification of the ages of nearly 12,000 children. See *Journal of the Royal Statistical Society*, Vol. 79, p. 309.

1. The reduced period gives a more definite measure of the mortality;
2. The census of 1911 gave numbers for individual ages instead of quinquennial or decennial age periods. It was possible therefore to group these in a more scientific way and the groupings adopted were ages 4 to 8, inclusive, 9 to 13, etc., instead of 5 to 9, 10 to 14, etc., as formerly.
3. Marital condition was given in tabulating the deaths of women during that period. Wives show lighter mortality rates than widows, and spinsters lighter than wives, except between ages 45 and 55, though in the latter case the differences are not very marked.
4. The rates of mortality for the first six years of life were calculated from the birth and death returns without reference to the census figures.
5. Healthy Districts Tables were discontinued in favor of Tables based upon the mortality in
   a. London;
   b. County Boroughs;
   c. Urban Districts; and
   d. Rural Districts.

These four groups represent approximately varying degrees of urbanization.

The census population in 1911 was approximately 17,400,000 males and 18,600,000 females. Under the general title of English Life Table No. 8, there are published the following:

1. Life table for the entire area of England and Wales—males and females respectively, with the usual mortality functions.

2. Life tables for females only according to marital condition—(*a*) Spinsters, (*b*) Wives. and (*c*) Widows.

3. Sectional life tables—males and females respectively—(*a*) Administrative County of London; (*b*) Aggregate of County Boroughs; (*c*) Aggregate of Urban Districts; (*d*) Aggregate of Rural Districts.

Throughout the greater part of life London shows lighter mortality than the County Boroughs; both of these show heavier

mortality than the Urban Districts, while the Rural Districts show the lightest mortality of all. Female mortality in London up to age 25 shows better than any of the others.

The effect of the emigration of young adults, and of immigration caused by the return of older British subjects from overseas; also the effect of old age pensions on the statements of age; deliberate misstatements by women, etc., are discussed with other minor criticisms in a review of the Tables J. I. A., XLIX., pp. 96–107. The suggestion is made to attempt elimination of such errors—a dangerous proceeding unless based on carefully ascertained facts.*

The notation was changed from the old system used in English Life Tables to the regular actuarial notation Text Book form. If Mr. King had done nothing in actuarial science except to standardize our notation all future generations of actuaries would be in his debt.

An interesting part of the report is the mortality shown by the various tables reduced to one common basis for comparison. The rate is lower at all ages and for both sexes by the English Life Tables No. 7 than by No. 6, and lower at practically all ages in No. 8 than in No. 7.

EXPECTED DEATHS PER ANNUM IN A POPULATION DISTRIBUTED ACCORDING TO THE CENSUS RETURNS OF 1911—AGES 0 TO 89, INCLUSIVE.

	Ages Last Birthday.	No. 6.	No. 7.	No. 8.
Males.....	0– 4	106,677	92,197	79,357
	5–59	122,830	101,995	91,628
	60–89	94,369	87,114	84,216
	0–89	323,876	281,306	255,201
Females.....	0– 4	89,394	76,879	65,980
	5–59	112,806	93,667	81,497
	60–89	108,344.	94,472	91,091
	0–89	310,544	265,018	238,568

*See further discussion by Mr. King and others. J. I. A., XLIX., p. 297 *et seq.*; also J. I. A., XLVIII, p. 207, on "Graduation of Ages."

U. S. Life Tables 1910.

These tables are the first of any scientific value prepared by the U. S. Government from Census returns. When the census of 1910 was taken the Bureau of the Census called into consultation a committee of the Actuarial Society of America and this committee gave general advice, both with reference to the taking of the census and the tabulation of the data. Although the recommendations were not in every case followed, the general results, and the tables now available, prepared under the supervision of Prof. James W. Glover, mark a notable epoch in the history of mortality investigations in the United States.

The tables, which were published in 1916, are based upon the census of the population taken in 1910 and the deaths during the calendar years 1909, 1910, and 1911 for the original registration states comprising: Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, Connecticut, New York, New Jersey, Indiana, Michigan, and the District of Columbia. Accordingly the tables do not exhibit the mortality of the entire area of the United States, but deal chiefly with the northeastern section.

The census was taken as of April 15, 1910. The data were entered on the schedules by the enumerators, and the people were recorded as of their usual place of abode. The work of enumeration commenced on April 15, and in cities had to be completed within two weeks; in country districts within thirty days. An estimate of the population was made as of July 1, 1910, being the central point of the three calendar years 1909, 1910 and 1911. The estimated population in these original registration states was 24,131,759, and the reported deaths for the three years were:

1909	353, 576
1910	377, 015
1911	368, 087

The area covered contained roughly one quarter of the population of the United States. In much of the remaining area the registration system is either faulty or practically non-existent.

Twenty-five tables were published—the first being a general

table dealing with both sexes and covering the entire area, the second and third being the same except for division into male and female; the next twelve for males and females separately in subdivided classes of the population—namely:

1. White,
2. Negro,
3. Native White,
4. Foreign-Born White,
5. Cities,
6. Rural.

The last ten tables deal with place of residence, giving male and female mortality separately in five individual states: Indiana, Massachusetts, Michigan, New Jersey and New York.

The lowest mortality rate amongst white males is shown at age 11—viz: 2.28 per 1,000; amongst white females, it is 1.98 per 1,000 also at age 11. The mortality amongst foreign-born white males is lighter than that of native white males between ages 21 and 38; but at other ages is heavier. This may indicate selection on the part of healthy immigrants and the immigration authorities. In general the mortality of men is greater than that of women. The rate of mortality in cities is much higher than in rural districts during practically the whole of life, and negro mortality is very pronouncedly higher than that affecting Caucasians.

The following tables give some indication of the relative mortality rates:

UNITED STATES LIFE TABLES—1910.

Death Rates per 1,000 of Population—i. e.: 1,000 q_x.

Age.	Native White Male.	Foreign-Born White Male.	Negro Male.	Native White Female.
10	2.37	2.47	5.02	2.06
30	7.14	5.80	14.96	6.13
40	10.02	10.53	21.03	7.76
60	27.21	36.81	50.79	22.06
80	132.43	141.76	131.27*	121.23
Corresponding Values of e_x.				
10	51.93	50.30	40.65	54.43
30	35.61	33.71	27.33	37.98
40	28.33	26.03	21.57	30.33
60	14.58	13.06	11.67	15.78
80	5.15	4.98	5.53*	5.47

* Ages probably exaggerated.

The tables dealing with different states indicate that mortality in Indiana and Michigan is notably lower than in the older communities of Massachusetts, New Jersey, and New York. This may partly be accounted for by the larger proportion of rural population in the former and the industrial centres with larger proportion of foreign born in the latter. Other sections of the United States are understood to show still lower death rates, while the South is not represented at all in the compilation herein described.

On account of the importance of infant mortality, and the rapid decline from birth during the first year, a separate table is given in conjunction with each of the life tables showing the rates of mortality and their derived values for each month during the first year. The calculations in most cases were based upon the enumerated population and reported deaths. This method is generally found unsatisfactory, yet in this instance was adopted in preference to using the birth registration statistics, which are also unreliable. The figures should be used with caution—they show significant differences in mortality conditions in the various classes of the general population, as, for example, the usual higher rate affecting male babies, and a very high rate in infancy amongst negroes.

From ages 15 to 85 the method of adjustment was that of osculatory interpolation with fifth differences, the ages being grouped in quinquennial sets 4–8, 9–13, 14–18, etc. For the first five years a more direct method was employed, and the interval from 5 to 13 was bridged over by ordinary fourth difference interpolation formulae. At the advanced ages Wittstein's formula was employed, the rate of mortality being taken as unity at age 115. This is an unusually advanced age for the limit of life, yet it finds precedent in the English Life Tables Nos. 7 and 8. In order to join the osculatory interpolation with the Wittstein Graduation, Spencer's 21-term formula was employed over a range, usually small, sufficient to insure a smooth junction.

In the explanation of the life tables, where the above summary of graduation methods is given, it is stated that powerful smoothing formulas were not used, as it was not always easy to distinguish the irregularities which are characteristic of the population from those which are merely due to defective enumeration and

it was therefore deemed better to present the life tables in an approximately unadjusted form. It is intended to publish all the original data in a later report with a detailed account of methods employed in constructing the life tables.

The tables are valuable as being the first of the kind prepared in the United States but in studying the difference in the various rates of mortality, those factors likely to produce differences should not be forgotten, as, for instance, in comparing the expectation of life of people living in the cities with those living in the country, it must be remembered that the cities contain a larger proportion of foreign born, while hospitals and sanitariums are mostly situated in towns and cities. The tables having been prepared from an estimated population and from reported deaths, the errors likely to have occurred in registration and tabulation, even as late as 1910, may be considerable. Where all the deaths are not recorded, as is generally the case in registration statistics, the error tends to decrease the death rate, and this factor may have caused part of the difference between rural and urban communities, since town and city registration is more complete than rural. The statutory obligation to register any death before a burial permit will be granted can be better enforced in city districts.

TABLES FORMED FROM THE EXPERIENCE OF LIFE INSURANCE COMPANIES.

Davies' Equitable Table.

The Equitable Society was formed in 1762, and is the oldest Life Office at present having a separate existence. Some years after its formation this Society adopted the Northampton Table as the basis of premiums, and Dr. Price recommended that the actual deaths experienced should be compared with those expected by the Northampton Table; this was accordingly done annually.

In 1800, Mr. Morgan, then Actuary of the Equitable, stated that the deaths among the members for the preceding thirty years had been as follows:

Ages 10–20	One-half of the number expected by the Northampton Table.		
Ages 20–30	One-half	do.	do.
Ages 30–40	Three-fifths	do.	do.
Ages 40–50	Three-fifths	do.	do.
Ages 50–60	Five-sevenths	do.	do.
Ages 60–80	Four-fifths	do.	do.

This table shows that the actual deaths were very much less than the expected, and Mr. Griffith Davies adjusted the Northampton Table to follow the experience of the society, his results being published in 1825. The methods he used are now unimportant; but he obtained the numbers living for each tenth age, and interpolated by finite differences for the intermediate ages. His table was not properly a mortality experience, deduced from the facts; but only a modification of the Northampton Table to obtain figures more nearly in conformity with actual conditions.

Morgan's Equitable Table.*

This table, published in 1834, was formed by Mr. Arthur Morgan, Actuary of the Equitable Life Office, from the experience of the society from its commencement in September, 1762, to January 1, 1829. It is the first table deduced on correct principles from the records of a life insurance company.

* J. I. A., Vol. XXIX, pp. 113–7, by James Chatham.

There were 21,398 lives, of whom 6,930 were living on January 1, 1829; 9,324 had surrendered, forfeited, or discontinued their policies; and 5,144 had died. Altogether there were 266,872 years of life under observation and the average period of exposure was 12¼ years.* The effect of the selection of lives was discussed and the rapid increase in mortality after date of entry was referred to.

The following shows the way in which the facts were tabulated:

Age at Entry.	Age Attained 30.				Age Attained 31.	
	Attained the Above Age.	Living on Jan. 1, 1829, at Above Age.	Discontinued their Assces, at Above Age.	Died.	Attained the Above Age.	Living, etc.
.	.					
.	.					
24	.					
25	421	10	29	2	380	etc.
26	.					
27	.					
	.					etc.
	.					

Thus opposite each age at entry under the Column headed "Age Attained" were put:

1. The number living who attained that age.
2. Those who were existing at that age on January 1, 1829, and who thus passed out of observation.
3. The number who ceased to be members at that age, and who thus also passed out of observation.
4. The number who died between that age and the next.

Under *Age Attained 30* opposite age 25, we find 421 in the column showing the number living, which is the number out of those entering at age 25 who continued members of the society until they attained age 30; of these, 10 were existing† on January 1, 1829, while 29 ceased to be members between the ages 30 and 31, and 2 died. Deducting 10 + 29 + 2, or 41, from 421, gives 380, the number who entered upon their thirty-second year of age, which was placed under "Age Attained 31" opposite 25, etc. Members were assumed to attain the age stated at entry (i. e., age *next* birthday) on the first of January following entry. This was therefore the first "Calendar Year" investigation.

* J. I. A., Vol. II, p. 202.

† The word "existing" is used with an unusual meaning, the letter E being used in mathematical formulas.

The table is now of historic interest only, and it is therefore unnecessary to describe the processes in full.

Actuaries' Table; Combined Experience Table; or Seventeen Offices' Experience 1843.

As it was felt that the rate of mortality among insured lives must be different from that of the general population on account of the former having to pass a medical examination, it was resolved at a meeting of actuaries and others connected with life insurance, held in London in 1838, to ask various companies to contribute their experience. A committee was formed and issued schedules in the following form to the contributing offices:

For Use of Office.	Current Age at Entry.	Year of		If by Death, D.	Sex, if Female, F.	Distinction into Town, (T). Country, (C). Irish, (I).	Cause of Death.	Special Risks and Remarks.
		Entry.	Exit.					

The forms were thus confined to a record of the *policies* issued; but the Equitable and Amicable Societies apparently furnished their observations on lives only, so the general results were mixed.*

It was believed that the rate of mortality would not be sensibly affected by the fact that a man might often effect several policies at various ages, and might at death be treated as several different persons. It has been said that this has the effect of diminishing the rate of mortality at the early ages, and increasing it at the old ages; but the extent of this tendency must be slight.

The returns embraced 83,905 policies, of which 44,877 were existing at the close of the observations (December 31, 1837), 25,247 had discontinued, and 13,781 had died.†

The committee state that the tables represent a lower rate of mortality than can be expected during a longer period of time than that over which the observations extended, for the average duration of all the policies was less than 8½ years, notwithstanding that the two oldest companies then existing (the Amicable and the Equitable) were included. The average duration of policies embraced in nearly one-half of the experience was under 5½ years. More than one-half of the policies effected were existing at the close of the observations, and nearly one-third had been discontinued.

*J. I. A., Vol. II, p. 203. †J. I. A., Vol. X, p. 197.

The tables were constructed by a "calendar year" process, and were adjusted according to a formula given by Mr. Woolhouse. The mortality amongst Irish lives was found to be heavy as compared with those resident in Great Britain. The mortality amongst assured females from ages 20 to 50 was considerably greater than amongst males of the same ages. From 50 to 70, it was less, while above 70 it was found to be sometimes greater and sometimes less, but at the advanced ages the statistics were quite inadequate to enable one to form a reliable opinion.

The Table was adopted as the standard for valuation purposes in Massachusetts about 50 years ago, and continued in very general use till 1901, as it was considered a "safe" table. Industrial policies were also valued on this basis, so it became necessary to extend the mortality rates down to age 0, and the National Convention of Insurance Commissioners in 1883 adopted the following rates for ages 0 to 9, recommended by D. P. Fackler, W. S. Smith, D. W. Whitney and A. F. Harvey:

Age.	l_x	d_x	q_x
0	143,400	22,184	.1547
1	121,216	7,697	.0635
2	113,519	4,030	.0355
3	109,489	2,617	.0239
4	106,872	1,892	.0177
5	104,980	1,428	.0136
6	103,552	1,123	.0108
7	102,429	922	.0090
8	101,507	792	.0078
9	100,715	715	.0071

INSTITUTE OF ACTUARIES TABLES PUBLISHED 1869— (H^m AND OTHERS).

In 1862, the Council of the Institute of Actuaries, aided by a joint committee of the Faculty of Actuaries and the Managers' Association (The Associated Scottish Life Offices), undertook to collect and tabulate the materials requisite for investigating the rate of mortality among insured lives, and 20 companies agreed to furnish their experience. The particulars of the lives were furnished on cards, which gave the following particulars, and 180,000 of which were sent in:

Policy No.	Healthy or diseased	Age at Entry	Cause of Death
Life	Year of Entry	Age at Exit	Remarks.
British, Irish or Foreign	Year of Exit	Mode of Exit	

There were 10 English and 10 Scottish companies contributing their experience, and on the Scottish card, the following particulars in addition to the above were given:

Sum Assured.
Class.
Exact dates, and
English, Scotch, instead of British.

The office age at entry (i. e., next birthday age) was inserted in the card, which afforded the means of approximating to the actual age by the assumption that the insured attained that age at the end of the year of entry.

The cards were first of all sorted into four divisions, viz.: (1) H^m, (Healthy Males); (2) H^f, (Healthy Females); (3) $D^{m,f}$, (Diseased Male and Female); and (4) those exposed to extra risk from climate, occupation, etc.

The cards in each of these divisions were then arranged according to the name of the life, with the view of bringing together all those relating to the same person and eliminating duplicates. The next step was to sort the cards in each division into three groups according as they had passed out of observation—(1) by death, (2) by discontinuance of their insurance, and (3) by surviving the period at which the observation closed, briefly termed "died," "discontinued," and "existing." All those of the same age at entry were next brought together, and finally arranged according to age at exit. The number of cards was reduced by elimination of duplicates to 160,426 persons, of whom 26,721 died, 45,376 discontinued, and 88,329 were existing at the close of the observations. The average duration of exposure was more than 9 years. Females formed about 11% of the total. The H^m table started at age 10, with a radix of 100,000.

The tables were formed by the calendar year method, it being assumed that the entrants in any year came under observation on the average at the middle of the year, and would attain the age at entry (i. e., age next birthday) at the close of the calendar year. This short period from the date of entry to end of the year of entry (assumed to be six months) was called the year "*O*"; and in keeping the mortality separate for each age at entry, as already explained, the mortality during this period was assumed to apply to the previous age. Thus, those who entered at age 30 next birthday, were assumed to be on an average exposed to risk for

six months in the year "*O*"; and the mortality for this six months was set down as applying to age 29. Later investigation showed that the average period of risk in the calendar year of entry was less than six months, also that the average true age at issue was less than six months younger than the age next birthday as stated.

The method of keeping the various ages at entry separate from each other was adopted in order that future investigators might make use of the facts in forming "select" tables, afterwards described. The committee made no immediate use of this arrangement, but combined in the H^m table all those of the same attained age whether they had been insured for only a year or two, or for many years. The effect of selection ceased to be of any practical importance after the expiry of 5 years, and accordingly the $H^{m(5)}$ table was formed, the experience of the first five calendar years after entry (approximately 4½ years) being neglected.

The effect of forming a table in the way the H^m was formed (known technically as an "aggregate" table, as opposed to a "select" table) i. e., entering new or select lives at each age with the general body of policy-holders at that age, some of whom must of necessity be unhealthy, has the effect of making the apparent mortality at the older ages higher than would be shown by a table taking account only of the new entrants at such higher ages, because the unhealthy lives are brought forward and included. The method has the opposite effect at the younger ages, because a body of lives, selected at a relatively young age, becomes "mixed" and contains a proportion of unhealthy lives; but the proportion of such unhealthy lives is reduced by the introduction of more healthy lives year by year, so that the lighter mortality amongst these newly selected entrants reduces the rate in respect of those who entered at the younger age. Premium rates computed from such a table are lower at young ages and higher at old ages than a select experience shows to be correct.

At ages under 45, the H^l table shows much heavier mortality than the H^m, but after that age lighter—thus confirming previous investigations. The tables were graduated by Woolhouse's formula.

These tables came to be very generally employed in Great Britain, and were also adopted as a standard for all life insurance purposes in Canada. They were never officially recognized and have been very seldom used in the United States.

The American Experience Table, 1868.*

The American Experience Table of Mortality, now recognized as the standard table in the United States, was formed by Sheppard Homans and was first published under its present name in a schedule attached to an Act passed by the legislature of the State of New York on May 6, 1868. The author never gave full particulars of the data employed. He used mortality statistics deduced from the experience of the Mutual Life Insurance Company of New York, but the figures were inadequate at the older ages and accordingly he arbitrarily adjusted the table. The table starts at age 10 with a radix of 100,000 and ends with three deaths between ages 95 and 96, the latter being therefore the limiting age of the table. It has been much used, and has grown in popularity because it has furnished a safe basis of measurement of American mortality amongst insured lives after the first effects of selection have disappeared.

At the first dinner of the Actuarial Society of America, April 25, 1889,† Mr. Homans said:

"The result was that after I had collated the experience of the Mutual Life I drew a curve representing the approximate rates of mortality at different ages; and then found by a simple method of adjustment the rates of mortality now called 'The American Experience Table'—a name, however, that was not given by me. The table has for its basis the experience of the Mutual Life; but it is not an accurate representation of that individual company. In other words, it is not intended to be, and never was claimed to be an accurate interpretation of the experience of the Mutual Life."

He further stated that the American Table was prepared simply as a study, and that he had made the limiting age 96 because in the records in different countries he could find no instance of any insured individual attaining the age of 100 years.

The Mutual Life experience, also prepared by Mr. Sheppard Homans, was published in 1858 and dealt with a period of fifteen years from the commencement of the company in 1843. Mr. D. P. Fackler has pointed out that premiums based upon the new tables were adopted in 1861 so that the modifications contained in the

* Walford's "Insurance Cyclopaedia." T. A. S. A., Vol. X, pp. 509–514.

† T. A. S. A., Vol. I., pp. 33–34.

American Experience Table as compared with the mortality rates of the earlier table of 1858 must have been made about the year 1860. They must therefore have depended on the additional experience of the Mutual Life for the years 1858 and 1859 only, or in other respects have been adjusted by Mr. Homans arbitrarily from a study and comparison of other tables. The new table "was intended to represent the death rate among insured lives residing in salubrious districts *after the effects of medical selection were eliminated.*"

The graduation of the Mutual Life Table of 1858 was by a graphic process, but no particulars are given of the method used by Mr. Homans in graduating the American Experience Table. Recent investigations* would seem to indicate that the following may be considered as established facts:

1. The American Experience Table is based on a table of the values of the reciprocal of q_x to three decimal places for each age from 10 to 95 inclusive. The method by which this table was derived from the original data is unknown.

2. From the values of the reciprocal of q_x the value of the logarithm of the rate of mortality was taken out to seven decimal places.

3. Starting with a radix of 100,000 at age 10, the value of $\log l_{10}$ was taken out to seven decimal places. The logarithm of q_x was added to it to form $\log d_{10}$. The natural number of d_{10} was extracted to the nearest unit and subtracted from l_{10} to form l_{11}, &c., the process being repeated for each age.

In 1902 Mr. Arthur Hunter made a very successful graduation of the American Experience Table by the application of Makeham's Law of Mortality, and this Makehamized Table is of great value in simplifying the calculation of benefits depending upon two or more lives.

The table is now generally prescribed by State Laws for valuation purposes and has been adopted almost universally in the United States.

*T. A. S. A., XII, p. 253; XIV, pp. 27-37, and 354-363.

Thirty American Offices' Tables.*

Tables were compiled in 1881 by Levi W. Meech from the returns of 30 American Offices, dealing with 1,027,529 lives, of which 549,418 were existing at the close of the observations (1874), 46,543 having died, and 431,568 having discontinued. The returns were furnished on cards, which gave:

> Residence, Amount, Class, Calendar year of entry,
> Calendar year of exit, Age at entry, Sex, Cause of death.

Male and female lives were investigated separately, and each subdivided into the three classes—existing, discontinued, and died. Less than 5% of the lives were female. These classes were again subdivided into durations of Policies, 0, 1, 2, 3, 4, . . . , and these again into still smaller groups according to the age at entry. Policies were assumed on the average to be taken in the middle of the initial year, and were treated as exposed for half of the first calendar year. Those who passed out of observation were also assumed to do so at uniform intervals. Average policy years thus begin and end with the middle of the calendar year, while the average birthday is assumed to be at the middle of the initial year, the general assumptions being similar to those made in forming the Institute of Actuaries' Table. The office age at entry was "nearest" birthday; and the entry ages were diminished by ½ for the period of exposure in the year of entry, i. e., as if the ages were changed from the middle to the beginning of the calendar year of duration. The average age at entry was 35.23 years, and the mean duration of the policies was 4.36 years. The mortality was investigated by lives, but the principal tables were based upon amounts insured and claims, with slight adjustments towards the extremes of the table. The rate of financial loss was greater than the rate of mortality, showing that the claim rate was higher amongst policies of large amount. The effect of medical selection was stated to disappear within 2½ years.

At ages under 45, the mortality of females was found to exceed that of males, indeed the excess from ages 20 to 35 was over 35%; from the age of 45 to 65 the order is reversed, though by a much smaller difference relatively, and again at the older ages (ages 67 to 95 inclusive) the mortality amongst females exceeds that amongst males.

* System and Tables of Life Insurance, by Levi W. Meech.

As 549,418 of the policies were existing at the close of observations, the record ceased before the real experience had developed. Had the investigations been deferred 15 or 20 years many more policies would have passed to their termination. The defect from having so many existing at the close of the observations was approximately supplied by *assuming* the existing to be carried forward until they should pass out of observation by discontinuance or death, the ratios of mortality and discontinuance for this purpose being taken as those actually experienced on Select Mortality and Withdrawal bases down to 1874. This was called the method of "Final Series";* of course it is effective only when the real and the assumed statistics are combined in an aggregate table, since it increases the proportion which the non-select lives at any age bear to the total under observation at that age.

The figures from 10 to 90 in the male table were graduated by Makeham's Law of Mortality, taking the ungraduated values of l_{25}, l_{40}, l_{55}, and l_{70} as data. Notwithstanding that previous tables showed considerable deviations from Makeham's Law, the results in this case showed close agreement with the ungraduated values, and this was explained by Mr. Meech as follows:

1. The observations were more numerous than those of any previous collection.
2. They were nearly homogeneous.
3. The statistics were accurately deduced.
4. The construction was improved by "Final Series."
5. The observations were free from misstatements of age common to census returns.

But as more recent experience has shown that many tables can be graduated by Makeham's Law, including even old tables like the "Carlisle," the suggestions as to the causes why this table should follow the law probably show a lack of familiarity with the adaptability and flexibility of the Law as afterwards developed. The graduation of the table for female life, on account of the small numbers above 70 was aided by comparison with larger collections, and then adjusted by Woolhouse's method. A very complete set of monetary values was published.

The statistics were also analyzed by states and territories of the United States, and tables were given of the actual and expected claims in each state or territory. These figures are less valuable

* See J. I. A., Vol. XXXII, p. 9 et seq.

than they would have been if the effect of selection had been taken into consideration, because in some states the business may be relatively recent whereas in others there may be much old business. In some of the Southern states the expected and actual claims are given for each county in the state separately—a desirable separation. There are also tables showing the causes of death from diseases which are grouped under the following heads: Zymotic, Constitutional, Nervous, Circulatory, Respiratory, Digestive, and Miscellaneous.

Standard Industrial Mortality Table.

Prior to 1907 the Policy Liabilities on Industrial Insurance were based on the same tables as were used for ordinary policies. Mortality tables had been compiled by individual companies from their own experiences and used in calculating premium rates; but as the statutes did not differentiate between the two classes the valuation of industrial policies was based upon the Actuaries, or Combined Experience for policies issued prior to January 1, 1901, and upon the American Experience Table for policies issued thereafter. Early in 1907, after modification of the New York laws, the Superintendent of Insurance fixed as the legal basis for New York State a table based exclusively upon the experience of industrial policies, called the "Standard Industrial Mortality Table"; and, while permission has since been granted by the New York Legislature to use other tables, this one is still recognized as a standard.

The Standard Industrial Mortality Table was based upon the experience of the Metropolitan Life Insurance Company during the ten years 1896–1905. The policies at risk and the death claims were obtained from data contained in registers in which were kept a record of all Industrial premium paying policies, classified according to year of issue and age at entry. The experience included 48,508,562 years of risk and 767,552 death claims.

The unadjusted data were first graduated by Woolhouse's formula for graduation from age 2 up to age 35 inclusive, also by Makeham's formula from age 20 to the end of life, using the groups of ages 27–39, 40–52, 53–65, 66–78 for obtaining the constants. The ages under 30 as obtained by the Woolhouse graduation were merged into the Makeham graduation by means of a second

Woolhouse grading, the resulting probabilities being retained from ages 12 to 35 inclusive, the Makeham graduation being followed from age 36 upwards. The original figures from ages 2 to 6 inclusive were retained without graduation, while for ages 7 to 11 an interpolation formula was used.

The table starts at age 2 with a radix of 100,000, and age 99 is the limit of life, there being 1 living at age 98. The rate of mortality q_x is lower than the American Experience from ages 10 to 21, then higher to age 87 inclusive; and at the very old ages it of necessity becomes lower again since the limit of life in the American experience is age 96. Medical selection is not such an important factor in case of industrial as of ordinary risks, since the applicants in most cases are subjected only to a medical inspection, and accordingly the tables were based upon the entire experience, thus being "aggregate" in form. The mortality curve does not agree at all closely with any known table; but probably comes nearer the English life experience than any other.

National Fraternal Congress Table of Mortality.

The fraternal organizations meet annually in convention, and at the National Fraternal Congress held in 1897 the feeling was expressed that the Actuaries' Table of Mortality which was then the standard table for old line life insurance companies showed higher mortality rates than was necessary according to the past experience of the better class fraternals. Accordingly a committee was appointed to look into the subject, and this committee made a report at Baltimore in 1898, accompanying their report by a table of mortality which was afterwards adopted as the National Fraternal Congress Table.

One of the principal objects in view was, as stated by the committee, to prepare a standard table which would bring out premium rates as low as practicable, apparently even taking into account the profit from lapsing, no surrender values being paid, since the committee reported that the problem in the preparation of the table "involved the basis of a minimum rate, *with the elimination of cash, paid-up, and extended values.*" The mortality rates are therefore low; yet a year later the Committee stated that many fraternal orders had had an even more favorable experience, although others may have encountered a higher death experience than the tables indicate.

Details of the number of lives exposed to risk and of the number of years' exposure are not given, the statement made by the committee being

"The experience examined and available embraced the mortality experience of the Old Line companies in the United States, England, Canada, and Australia; of the Fraternals, the experience of the two oldest and largest in this country."

Apparently, therefore, the table consists of the arbitrary judgment of the members of the committee; this judgment was confirmed by a continuing committee one year later, the report in 1899 stating that the committee "has had opportunities for extending its research somewhat, as new data have been compiled of the actual experience of the fraternal orders, members of this Congress, which were not available one year ago." After a careful investigation nothing was disclosed which would lead the committee to recommend a change.

The two "oldest and largest fraternals in this country" at that time were The Royal Arcanum and the Ancient Order of United Workmen, "and it is generally understood that the experience of the Royal Arcanum largely controlled in fixing the data agreed upon by the committee." The table was tested still further and a voluminous report submitted to the National Fraternal Congress in 1906, when the congress considered that it was justified in continuing its endorsement as originally given.

The table was graduated by Mr. George D. Eldridge, who states that the material was placed in his hands "late on the afternoon of November 16, 1898, the day before the committee having the matter in charge was to report to the congress then in session. The work of graduating q_x was completed during that night." The table was graduated on the assumption that q_x could be represented by a function of the form $A + Bc^{x+\frac{1}{2}}$; and as a preliminary step the value of q_x was determined experimentally from the material at certain ages "taken at fifty or over, with a view to dealing first with that part of the table only which, under the general limitation of fraternals, is not affected by the admission of new lives." From these experimental values of q_x the three constants, A, B, and c were obtained. These constants gave a rate of mortality at age 20 of .0052511; but it was the judgment of the actuaries that, excluding the savings from selection during

the three years immediately succeeding admission, a rate at age 20 of .005 was a far more justifiable minimum. The latter was therefore adopted and a subtractive series formed from ages 20 to 45.

It will be observed generally with reference to the above table that while the results may be practical as a standard of measurement from the fraternal standpoint, representing *minimum mortality rates* for adoption by such orders, nevertheless the processes adopted in deducing the table were largely empirical.

SELECT LIFE TABLES.

Insurance companies do not accept all the applicants who offer themselves, but reject a small proportion of the lives proposed for insurance, taking the great majority at the usual premium rates, charging others an increased premium, and declining entirely a few whose prospects of longevity appear to be much below the average. The necessity for this distinction arises from the fact that if the benefits of insurance were free to all comers, there would be an undue proportion of under-average lives, more than the normal proportion in the population, since an unhealthy man needs insurance more than a healthy one. Moreover the healthy would with reason object to be classified with the diseased and to pay the same premium rates. Selection is effected by means of medical and other reports on the health, family history, personal history and habits of the individual whose life is proposed for insurance, with due regard also to occupation and habitat.

For the first two or three years after entry the effect of this selection is quite apparent, the death rate amongst recent entrants being very low as compared with the rate amongst those who have been insured for several years, or as compared with the general population; but, as years pass the rate of mortality approaches nearer to that of non-select lives, although it is the general opinion that the effects of the first selection never entirely disappear. Mr. Sprague held* that the mortality among select lives gradually increased until it attained a maximum, after which it diminished; this he attributed to two counterbalancing causes:

1. The selection as exercised by the Office when insurance is effected, and
2. The selection exercised by the lives insured in having the right to withdraw, and the consequent withdrawal of a considerable proportion of the healthy lives during the early years of insurance.

More recently† it has been urged that withdrawals do not have the effect of reducing the proportion of healthy lives; indeed the

* J. I. A., Vol. XV, p. 328.

† J. I. A., Vol. XXIX, p. 81; also XXXII, p. 117, etc.

direct contrary is sometimes accepted on the ground that withdrawal from a company in good standing is more frequently a result of financial embarrassment or irregular habits. The opinion that discontinuances have not an adverse influence on the mortality seems to be gaining ground—even Mr. Sprague stating that he had "considerable doubt" as to whether the lives which withdraw are on the average better than those which remain. Until the point is settled by convincing statistics, which are difficult to procure, it remains a matter of opinion with plausible arguments on both sides.

To investigate the effect of selection properly, it is desirable to trace a large number of entrants from each age at entry until they all pass out of observation; this has been done on several occasions, the first important investigation being made by Mr. J. A. Higham prior to the year 1850. In that year he read a paper before the Institute of Actuaries on "The Value of Selection among Assured Lives, etc." This consisted of an analysis of the experience of the Equitable Society. In 1851 he submitted another paper on "The Value of Selection as Exercised by the Policy Holders against the Company."* His deductions were based upon the Actuaries or Seventeen Offices' Experience, but his figures were not much used for practical purposes. Mr. Higham said "the probability of surviving the year immediately after selection is a quantity which we have not at present the means of measuring"; and the means of measuring this quantity accurately was not obtained until about the year 1900.

After the observations from which the H^m Experience Table was compiled were published, Messrs. King and Sprague devoted themselves to a careful analysis of selection and the monetary tables published by Sprague in 1879 were widely used for the succeeding twenty years.

King's Analysed Tables—1876.†

The first important discussion of the rate of mortality by duration of insurance after publication of the H^m mortality experience was opened by Mr. King in a paper "On the Mortality amongst Assured Lives," read before the Institute in 1876. This paper

* J. I. A., Vol. I, p. 179.
† J. I. A., Vol. XIX, p. 385.

marked an epoch in the study of mortality statistics. Extracts from Mr. King's own description of his new mortality tables will give an indication of his procedure:

"I have prepared and graduated ten mortality tables for ages at entry 20, 25, etc., up to 65. They were formed from the tables of 'exposed to risk' and 'died,' . . . by tracing separately . . . those who entered at the ages 20, 25, etc., respectively. It was of importance to include as many of the observed facts as possible; I have, therefore, after the first ten years of assurance, combined with those entering at age x those also entering for ages $x-2$, $x-1$, $x+1$ and $x+2$, thus making x the central age of a quinary group. . . . It would have been more satisfactory had larger numbers enabled us to dispense with this grouping; but, after all, it can make very little difference in the mortality among persons of the same age, when they have been assured for some considerable time, whether they entered at a period two years more or less remote. Two slight errors are introduced which tend to neutralize one another.

"In the early years of assurance, however, the case is different. During the first ten years, the medical examination at entry must exercise a decided but rapidly decreasing influence, while the selection against the company caused by discontinuances is at its strongest. It is, consequently, of importance to avoid mixing those who have been assured, say 5 years, with those who have been assured 6 and 7 years, or only 3 and 4. The numbers, moreover, are sufficient to render it unnecessary; and for the first ten years of each table, I have brought into account the exposed to risk and the deaths for that age at entry only which appears in the heading. The table for age 65 is the only one where the numbers were so small as to produce serious fluctuations; but for the sake of uniformity, I have adhered to the same plan throughout."

At the advanced periods of life, commencing after age 60, Mr. King combined the observations for several ages at entry:

Entry ages 20 and 25	being	combined	at	age	60.
" " 20 to 45	"	"	"	"	80.
And all entry ages	"	"	"	"	89.

The tables were adjusted by a very simple method of graduation, viz.:

$$d_x = \frac{d'_{x-2} + d'_{x-1} + d'_x + d'_{x+1} + d'_{x+2}}{5}$$

where d' represents ungraduated and d the graduated number of deaths. When the result was still too irregular the operation was repeated.

Mr. King assumed that the rate of mortality for the fraction of the year of entry, "year O," would continue for a full year,

adding "perhaps by so doing the mortality is slightly underrated." He used the mean of 3 ages at entry, thus obtaining a simple and rough graduation of the first years mortality.

Mr. King also analysed the rate of discontinuance which "decreases not only with the duration of the policy, but also with the age at which the policy was effected." The rates of discontinuance at age 35 and at all ages combined were as follows:

Rate % of Discontinuance.			Rate % of Discontinuance.			Rate % of Discontinuance.		
Year.	Age 35.	All Ages.	Year.	Age 35.	All Ages.	Year.	Age 35.	All Ages.
0	2.5	2.7	5	2.7	2.8	10–14	1.4	1.4
1	6.2	7.0	6	2.2	2.4	15–19	1.0	.9
2	4.2	5.0	7	3.2	3.6	20–24	.8	.7
3	3.8	4.1	8	1.9	1.8	25–29	.4	.5
4	2.9	3.3	9	1.5	1.6	30 and over	.4	.4

Sprague's Select Mortality Tables.*

In 1879 Mr. T. B. Sprague published the results of an investigation he made from the H^m statistics to determine the effect of selection, and to show the uses to which Select Tables could be put. He first reduced the figures for each age at entry to a common radix of 100,000; and, in order to lessen the irregularities arising from small numbers, he grouped the entrants at five ages to get the rate of mortality for the central age of the group.

The same assumptions were made as to ages at entry as were made in compiling the Institute Experience, namely, that the entrants at any age next birthday attained that age at the end of the year, also that the initial period, "year *O*," covered half a year. To deduce the mortality for age 30, the five year group contained:

100,000 of exact age 29½ } which gave 200,000 of mean age 30.
100,000 of exact age 30½ }

100,000 of exact age 28½ } which gave 200,000 of mean age 30.
100,000 of exact age 31½ }

50,000 of exact age 27½ } which gave 100,000 of mean age 30.
50,000 of exact age 32½ }

500,000 of mean age 30.

The reason for taking only 50,000 at the two last named ages was of course that the balance of 50,000 in each case entered into the adjacent groups; i. e., the other 50,000 at age 27½ were included in the group for age 25 at entry, and the 50,000 at age

* J. I. A., Vol. XXI, p. 229 et seq.

32½ in the group for age 35. By reducing the figures to a common radix, the facts at each age had the same weight assigned to them in the final table.

The following table indicates the method and gives the figures used for age 30:

Time Elapsed.	Assumed Age at Entry.						Average Age at Entry 30.		
	27½	28½	29½	30½	31½	32½	Survivors.	Total Deaths.	Age Attained.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Years									
0	50,000	100,000	100,000	100,000	100,000	50,000	500,000	0	30
½	49,962	99,788	99,931	99,754	99,676	49,864	498,975	1,025	30½
1½	49,550	99,346	99,394	99,316	99,115	49,609	496,330	3,670	31½
2½	49,167	98,655	98,644	98,594	98,347	49,256	492,663	7,337	32½
3½	48,672	97,940	97,490	97,839	97,573	48,818	488,332	11,668	33½
4½	48,345	96,823	96,166	96,926	96,841	48,315	483,416	16,584	34½
5½	47,986	95,858	95,142	96,024	95,960	47,860	478,830	21,170	35½

This table gives the numbers living at the end of ½, 1½, 2½, etc., years; and in order to obtain the numbers at the end of full years, Mr. Sprague divided the deaths in Column 9 of the above table into half-years by a method of differences. The deaths in the first and second half-years were then added together and divided by the corresponding numbers exposed to risk to give the rate of mortality for the first full year, and so on for subsequent years.

The tables were computed on the further assumption that the $H^{m(5)}$ table represented the ultimate rate of mortality. Accordingly having deduced the Select rates for the first five years, and having graduated them by the Graphic Method, these graduated rates were joined to the $H^{m(5)}$ rates. When the former exceeded the latter in the years one, two, three or four, the $H^{m(5)}$ rates were adopted from that year onwards; after the first five years the $H^{m(5)}$ Table was adopted in all cases. This saved labor in calculations, because all the more useful values had been published for the $H^{m(5)}$ Table, and those for the early years only had to be added.

From the rates of mortality for quinquennial ages, found as above shown, the rates for intermediate ages were obtained by a process of osculatory interpolation.

These tables quickly came into general use, especially for the calculation of premiums; and, as compared with ordinary life net premiums under the H^m Table, those by the Select Tables are higher up to age 43, but after that age they are lower. King's Analyzed Tables gave similar results. As the method of computing the rate of mortality for the first year introduced a function involving the second year's mortality, the rate deduced by Mr. Sprague was probably too high, while Mr. King's method left the same factor avowedly too low.

In addition to the painstaking work of Mr. Sprague in this investigation there were four original ideas introduced by him at that time which deserve special emphasis:

1. The facility obtained by using a *Common Radix;*
2. The happy *combination* of his figures with the $H^{m(5)}$ *Table;*
3. The use of a new formula of *Interpolation;* and
4. The development of an excellent *Notation* for Select Tables.

Moreover, Mr. Sprague, in an elaborate and very valuable paper* showed the many uses to which select tables could be put whereby information could be obtained on points regarding which nothing could be learnt from the old aggregate tables. Those uses opened a new field of vision to many actuaries, and enabled them to solve problems, which some had scarcely even ventured to guess at previously. The result was the immediate acceptance by nearly all actuaries of the Select Table Principle.

British Offices' Life Tables, 1893.†

A growing opinion that the mortality of insured lives had developed on lines which made the H^m Table obsolete, with a strong desire for more truly representative tables, especially Select Tables, led to the formation of a Joint Committee of the Institute and the Faculty of Actuaries to compile new statistics. In 1894 cards were issued to the contributing offices, and sixty sent in returns. Males and females were dealt with separately by means of colored cards. Rated up lives and those subject to extra hazard were excluded, the experience being confined to healthy lives resident

* J. I. A., Vol. XXII, p. 406.

† British Offices Life Tables—several volumes published by C. & E. Layton, London. See especially "An Account of the Principles and Methods, etc.," 1903.

in Britain when the policies were issued. If extra premiums were *afterwards* incurred for foreign residence or occupation, the lives were continued under observation without modification on this account. The experience extended from the Policy Anniversary in 1863, or subsequent date of entry, to the anniversary in 1893 or previous exit. One of the objections to the H^m Table was thus avoided, namely—that the data for that table extended back to the earliest history of the oldest companies when sanitary and economic surroundings were entirely different.

The rates of mortality are shown separately for different classes of policies, as follows:

	Symbols Used.	No. of Years of Experience.	No. of Lives.	Approximate Mortality Ratio.*
Whole Life PARTICIPATING.....	O^M	7,056,863	551,838	100
Whole Life NON-PARTICIPATING	O^{NM}	602,591	56,807	110
Endowment.....	O^{EM}	897,673	132,043	75
Limited payments.....	O^{LM}	410,251	36,839	81
Increasing scale premiums.....	O^{IM}	207,709	23,280	109
Temporary.....	O^{TM}	36,489	11,603	
Contingent.....	O^{CM}	15,586	3,482	
Joint Lives.....	O^{JM}	90,171	9,195	

Select Tables are expressed $O^{[NM]}$, $O^{[M]}$, etc.; while for female lives the letter *f* is substituted in the above symbols instead of *m*.

Select mortality tables have been accurately formed for the first time and show the effect of the company's and the policy holder's selection. The effect of selection is shown to be of little importance after 5 or 6 years yet unadjusted tables are given which exclude:

1. The first 5 years;
2. The first 6 years;

etc., etc., up to the first 10 years.

By excluding the first ten years we probably get the *ultimate* rate of mortality. Select values during the first ten years have

* This "Approximate mortality ratio" is obtained by comparing the actual deaths between the principal insurance ages 30–50 with the expected deaths by the $O^{M(5)}$ table. The select period (first five years) was excluded in making the comparison. In the last three classes the non-select experience is scarcely sufficient to make a satisfactory comparison.

been graduated to join this ultimate table. Those tables which exclude a portion of the data have been called truncated—a word which does not seem expressive for the particular purpose in view.

Duplicate lives were eliminated from the aggregate tables; but all except simultaneous policies were retained in the select statistics, since two policies taken at different dates on the same life do not come into the same section of a select mortality table. Accordingly the Select Tables purport to deal with a larger number of lives and a greater number of years of exposure. In the aggregate and ultimate tables if two policies appeared on the same life, but with an uninsured interval between the discontinuance of the one and the issue of the other, the first was treated as withdrawn and the second as a new issue.

The O^M and the $O^{M(5)}$ as compared with the American Experience Table show the following results:

Age.	Complete Expectation of Life			Rate of Mortality q_x per 1000.		
	American.	O^M	$O^{M(5)}$	American.	O^M	$O^{M(5)}$
20	42.20	43.68	42.39	7.81	4.04	6.52
30	35.33	35.57	35.06	8.43	5.95	7.47
40	28.18	27.86	27.67	9.79	9.15	9.78
50	20.91	20.61	20.52	13.78	15.04	15.45
60	14.10	14.07	14.04	26.69	28.87	29.21
70	8.48	8.71	8.71	61.99	62.07	62.19
80	4.39	4.84	4.84	144.47	138.44	138.50

The rate of mortality q_x by the American Table is higher than the O^M at ages under 43, then considerably lower from that age to age 70, after which it is again higher. Net premiums for Whole Life insurance follow generally the same course; but at the principal entry ages 25 to 40 there is never so much difference as $1.00 per thousand. At age 25 the American Experience rate is 87 cents *more* than the O^M and at age 40 it is 49 cents *less*. The Expectations of life by the American Experience and the $O^{M(5)}$ show a remarkable agreement while annuity values, premium rates, etc., also differ but slightly.

Female Experience is given for: (1) Ordinary Life participating, (2) Ordinary Life non-participating, and (3) Minor Classes. As compared with male mortality the same characteristics appear as

had been noted in earlier investigations; there is also an indication of general superiority of female over male vitality; but medical selection does not appear to be quite so effective. A full analysis of female in comparison with male mortality was made by Mr. C. W. Kenchington* and a further investigation was afterwards made by Mr. A. J. C. Fyfe.†

Amongst the "Old Assurances" in force in 1863, there were comparatively few recent policies, and the method whereby such Old Assurances were included, while extending greatly the experience at the older ages, had the effect of introducing an increased number of non-select and aged lives into the aggregate O^M experience, as well as into the select experience. This would make the mortality curve of the aggregate table unduly steep after middle life; it probably accounts for the high reserves by this table. Although the H^m net premiums are lower than Sprague's Select up to age 43, the O^M are lower than the $O^{[M]}$ Select only up to age 29 —a result possibly arising from the same cause.

Net premiums by the O^M Table are less than those by the older H^m Table, but policy values average greater, a condition caused by the different slope of the mortality curve. The mortality in the various classes confirms the belief that the larger the rate of premium paid, the lower the mortality—an effective illustration of the selection exercised by policy holders.

A careful study of the new statistics as compared with earlier compilations appears to show:‡

1. A marked improvement in vitality;
2. The influence of medical selection is more persistent, and especially conspicuous amongst younger men;
3. Self selection by annuitants had not improved to the same extent as medical selection of applicants for insurance.

One of the new features of this experience is the publication for the first time of complete tables of withdrawals. These are given for ages at issue grouped around quinquennial points and for each year of duration from 0 to 9 inclusive, also for "10 and upwards."

The rates of discontinuance per cent. according to the O^M experience§ were as follows:

* J. I. A., Vol. XLIV, p. 105.
† T. F. A., Vol. VII, p. 21.
‡ T. F. A., Vol. IV, p. 82.
§ See J. I. A., Vol. XXXVII, p. 463.

Year.	Central Age at Entry.				Year.
	25	35	45	55	
1	3.05	2.42	2.25	1.98	1
2	8.59	6.56	5.70	5.17	2
3	5.81	4.44	3.88	3.46	3
4	4.21	3.40	2.92	2.64	4
5	3.27	2.66	2.44	1.95	5
6	2.78	2.34	2.09	1.77	6
7	2.30	2.08	1.70	1.41	7
8	2.17	1.72	1.49	1.11	8
9	1.91	1.67	1.33	1.02	9
10	1.68	1.52	1.21	1.11	10

The rates of discontinuance per cent. on *non-participating* whole life policies differed materially from the above as is shown by the following rates for central age at issue, 35, that is for the group of ages 32–37.

RATES OF DISCONTINUANCE PER CENT, O^{NM} EXPERIENCE.
Central Age 35.

Year.	Rate.
1	4.36
2	10.13
3	7.36
4	6.57
5	4.90
6	3.88
7	3.20
8	2.72
9	2.49
10	2.31

The graduation of the entire experience was placed in charge of Mr. George F. Hardy; and the $O^{M(5)}$ Table was graduated by Makeham's Law, constants being determined by a new method somewhat similar to the method of moments. In the adjustment of the O^{M} Table the use of Makeham's formula alone was not practicable; but the graduated table was built up on the basis of the graduated $O^{M(5)}$ by a double frequency curve connecting the two tables. As it was necessary to abandon or modify Makeham's formula the only object in view was to obtain a perfectly smooth curve for the O^{M} Table, and to represent as nearly as possible the ungraduated facts. But Makeham's Law was applied to the graduation of the $O^{NM(5)}$ experience,* and was found to give good results from age 20 onwards.

* J. I. A., XXXVIII, p. 501.

An ultimate table, excluding the first ten years of the participating experience, was graduated by Makeham's Law and the select values were graduated to join this Ultimate Table by an extension of the same law, introducing a function dependent on the duration of the insurance.

Specialized Mortality Investigation.*

This is not strictly a mortality table but rather an investigation into the relative mortality as affected by certain special features, such as: nationalities resident in the United States, occupations, personal and family histories, and localities. The work was undertaken in 1901, under the direction of the Actuarial Society of America, from thirty years' experience (1870–1899) of thirty-four life insurance companies in the United States and Canada. Its object was to supply those officers who have to decide upon the acceptance of risks with material for ascertaining whether a particular class is better or worse than normal.

The investigation was based upon policies except that if several policies were issued on one application in the same year only one, the longest in force, was reported. The duration of policies cancelled was computed by subtracting the calendar year of issue from that of termination, and of policies in force by taking the exact duration at the anniversaries in 1900.

The mortality experience in the different classes was compared with a table adopted to represent Standard Mortality amongst healthy lives. The expected deaths were computed by such Standard Table, which was based upon Farr's Healthy English Male Table, modified for ages 15 to 21, inclusive, and for 52 to 61, inclusive. For ages 15 to 21, eleven-tenths of the annual mortality of the then new male experience $O^{m(5)}$ was taken, while at ages 52 to 61, five-sixths of the $O^{m(5)}$ mortality was adopted. In order to allow for the effect of selection during the early years, the following percentages of mortality by the Standard Table were used:

Ages at Entry.	Percentages of Standard Mortality to Allow for Selection.					
	Year 1.	Year 2.	Year 3.	Year 4.	Year 5.	Year 6 and After.
15 to 28	45	64	79	90	97	100
29 to 42	50	68	82	92	98	100
43 to 56	55	72	85	94	99	100
57 to 70	60	76	88	96	100	100

* Volume published by A. S. A., 1903, 479 pp., folio.

The final results were published in 1903, with the actual and expected deaths for each class in the four age groups above shown as well as in total. The detailed facts were also published to permit of individual study, and from which more accurate conclusions may be deduced.

The classes of risks may be briefly described as follows:

1. *Policies for large amounts;* one class over $20,000.
2. *Policies granted on terms other than applied for;* 2 classes.
3. *Nationality;* divided into 4 classes.
4. *Occupation;* divided into 35 classes and covering army, navy, and marine service; the more important hazardous trades; liquor dealing; and railway service.
5. *Personal Disability;* covering 32 classes, including past history of diseases such as gout, blood-spitting, etc., unusual weights and unusual heights.
6. *Family history unsatisfactory;* covering 2 classes, dealing respectively with cancer and insanity.
7. *Place of Residence;* 22 classes, each relating to a different county in the United States, principally in the South.

Care must be exercised in using the results for the following reasons: All the lives investigated were *accepted* by companies and therefore may not show *average* results in some of the specialized classes. While those who were charged an extra solely on account of occupation were included in the experience, no risks were included if they had been treated as under average on account of personal or family history. Accordingly all the classes in the latter category may be viewed as the better selected risks of each class. Again, the standard of measurement (Farr's Table Modified) has not been universally accepted as representing the normal risk. If the standard mortality were too low for any group of ages the result would be to give apparently unfavorable results for those ages and vice versa.

Medico-Actuarial Mortality Investigation.

The Association of Life Insurance Medical Directors and the Actuarial Society of America felt that the work of the Specialized Mortality Investigation should be extended, and a joint committee was formed for the purpose, October, 1909; memorandum of instructions issued May, 1910. Forty-three companies supplied

the basic information and the work of compiling, tabulating, and recording the data was conducted at a central bureau in New York City.

The experience covered about 93 per cent. of the policies issued in the United States and Canada by all legal reserve companies during the years 1885–1909, traced to policy anniversaries in 1909. It dealt with the following:

a. 68 groups involving occupational hazard;
b. 76 groups of medical impairments;
c. 4 groups of women;
d. 3 groups of colored risks;
e. 4 groups of joint life policies;
f. A study of the influence of build on longevity.

Policies which had been treated as sub-standard for medical reasons were excluded; policies rated as sub-standard for family history alone, or for occupation, were included. The experience was based on policy years, and mean durations; in case of death the duration was curtate, thereby placing each death in the proper policy year.

To investigate the influence of build on longevity, the companies furnished records of their business on standard lives for one month in each of the sixteen years, 1885–1900, January being taken in the odd years and July in the even years, by this means securing an average of summer and winter weight. The mortality rates from this data were much lower than the standard table assumed in the Specialized Investigation. They were however confirmed by the experience of two large companies, and were therefore used for purposes of comparison. The rates of mortality were low at the younger ages, the ultimate mortality for the eleventh and succeeding policy years being less than 60 per cent. of the American Experience for attained ages below 40. After the first year the mortality by policy years for entrants under 50 showed the effects of selection to be relatively slight.

The committee asked the companies to furnish additional data to examine further the improvement in mortality and a standard table was thereafter formed. The following shows the extent of statistics:

Years of Issue.	Number of Entrants.	Total Number of Years' Exposure.	Deaths.	Expected Deaths.	Ratio.
1885–1892	80,976	781,852	7,180	6,850.03	105%
1893–1900	148,995	1,106,316	8,000	6,911.45	101%
1901–1908	270,404	926,108	5,042	5,441.91	93%
Totals	500,375	2,814,176	20,222	20,203.39	100.3%

The final table shows the mortality rates for the first four years of duration and thereafter is merged into a general table excluding these years,—now commonly mentioned as the M. A. Table. Only the first two years showed a distinctly lower mortality than the ultimate. The Committee was satisfied that the table might with confidence be used for the particular purpose for which it was constructed.

Subdivision of the data according to years of issue showed a continuous improvement in mortality. The committee issued a warning against the general use of this table for any purpose other than that for which it was prepared. It is based upon policies and should not be applied to the solution of financial problems, since mortality rates are higher when based upon amounts insured.

Five volumes of statistics were published from 1912 to 1914, the first volume containing details of the investigation and basic facts for the investigation of build, also the adjusted mortality table. Tables of average height and weight were prepared and in Volume II are published mortality ratios in groups of lives according to the extent of departure from the average build.* These tables should be used with caution, as the preponderance of recent business has the effect of indicating a lower mortality than normal in some cases, in others a higher. In addition to showing the effect of build on mortality Volume II contains an analysis of the causes of death amongst men at different ages; also tables of mortality amongst women† divided into four classes—

a. Spinsters;
b. Married women, beneficiary husband;
c. Married women, beneficiary other than husband;
d. Widows and divorcees.

* T. A. S. A., Vol. XV., p. 315, and XVII., p. 17.
† See also T. A. S. A., XVIII., pp. 318–326.

Classification by plan shows that endowments are favored by unmarried women, whereas life and limited payment forms are more often taken by married women, including widows and divorcees. The volume closes with statistics of mortality amongst North American Indians and certain colored groups. The mortality amongst negroes was shown to be about 40 per cent. in excess of the expected in spite of the careful selection of such risks.

Vol. III. deals with the "Effect of Occupation on Mortality," the most important divisions being the Liquor Industry, Railroad, Metal Trades, and Mining. The causes of death are not given.

Vol. IV contains statistics of cases showing medical impairments, including the effects of alcohol, also diseases affecting the various physical organs, the arteries, etc., including certain surgical conditions. It must be remembered that cases accepted without rating, although coming within these classes must have been unusually good in every respect except for the particular impairment. High mortality appears among those who have suffered from syphilis, even after two years' continuous treatment and one year's freedom from symptoms; also in cases in which albumen or sugar were found, cases of alcoholic habits, high pulse rate, abdominal girth greater than chest expanded, etc.

Questions of family history (especially history of tuberculosis), habitat mostly in southern states, malaria, and joint life insurance, are covered in Vol. V. It was developed that under joint policies to men and women, there was a high mortality among women at the younger ages, but at the middle and older ages they appeared better than men.

Students should read with close attention the warnings contained in "An Interpretation of the Results of the M. A. Investigation," T. A. S. A., Vol. XV., pp. 62–76.

Experience of Canada Life Office.

Published 1895, covering 46 years, 1847–93.

One of the principal objects of this investigation, made by Mr. Frank Sanderson, was to find out the effects of selection in the northern half of North America. All lives rated-up, or charged an extra, as well as female lives, were excluded. The investi-

gation was made according to the lives assured, and particulars were taken out on cards of somewhat similar form to those of the Institute 1893 Experience.

The policy year method of investigation was adopted, but some slight complication arose in regard to the ages on account of the next-birthday age having been inserted in the cards. The experience was reduced from fractional to even ages by means of the formula

$$l_x = l_{x-\frac{1}{3}} - \tfrac{1}{3}d_{x-\frac{1}{3}},$$

it having been found from actual experience that on the average the assurances were taken one-third of a year prior to the attainment of the age next birthday. But this assumption was ingeniously avoided in graduating the aggregate tables, constants having been determined from the facts at fractional ages.

In filling in the mode of exit four subdivisions were used: (1) Existing, (2) Matured, (3) Withdrawn, and (4) Died. The matured contained expired term assurances and endowments. The existing were carried to the anniversaries in 1893, and the duration was found by subtracting the year of entry from 1893, thus giving an integral number of years in all such cases. The nearest integral duration of exposure was adopted for the Matured and Withdrawn, while those who died were carefully located in the policy year in which death took place and assumed to be under exposure to the end of that year. Duplicate lives were eliminated, it would appear, from the select experience as well as from the aggregate.

The total number of entrants was 35,287 covering 296,481 years of life. 55% of the entrants were "Existing" in 1893; 37% had "Withdrawn" and "Matured"; and 8% had "Died." The average age at entry was 32; the average duration nearly 8½ years. Of the total years of exposure, more than one-half relate to ages below 40 and four-fifths to ages below 50, a fact that probably accounts for the small percentage of deaths for so high an average exposure, although for the "Died" alone the average duration was 13.55 years.

The tables deduced include an Aggregate Table, a table excluding experience of first five years, some Select Tables, as well as full particulars under each age at entry of the exposed to risk and deaths for each year of duration. Statistics are given of the

withdrawals, and it is shown that nearly half of these took place at the end of the first year. Makeham's formula for graduation was used for the aggregate table, the method of determining the constants being that introduced by King and Hardy; $\log c = .0425$.

In forming select tables the experience after five years was first investigated, and connecting mortality rates were formed during the first five years between the ages of 20 and 50 in the following manner: The first year's mortality was investigated by itself in three groups of ages, and then graduated for each age. The third year's mortality was found by combining the second, third and fourth years, the resulting mortality rates being graduated by Makeham's formula; and lastly, the second, fourth and fifth years' mortality were got by interpolation.

In the published volume there were included comparative tables showing the death rate as compared with that of the more important mortality tables in individual offices and groups of offices. The mortality of the Canada Life appears to have been exceedingly favorable, being practically the same as that of the Australian Mutual Provident. Statistics deduced from census returns would indicate that the conditions in Canada are peculiarly favorable to longevity. The satisfactory experience of the Canada Life office may be caused to a considerable extent by climatic influences, combined with strict medical selection and a goodly proportion of risks from rural communities.

D and N commutation columns and annuity values at 4% interest are given, and it is found that the reserves by this table are high as compared with the standard tables.

The government of the Province of Ontario adopted the experience as the basis for the calculation of minimum rates for fraternal societies.

A separate investigation was made into the mortality amongst substandard lives, but the paucity of data and combination of various classes of risks renders any conclusions from this class of doubtful practical value.

Mortality Experience of Gotha Life Office, 1829–1896.*

This investigation, made under the supervision of Dr. Johannes Karup, embraces the experience of the Gotha Life Office on all classes of life and endowment insurances from 1829 to 1895; the observations ceasing on the policy anniversaries in 1896. The

* This account has been taken from T. F. A., Vol. V, p. 87 et seq.

policy year method was used and there were 150,594 lives with 2,255,813 years of exposure, and 46,480 deaths. All joint life, survivorship, and short term insurances were excluded. The experience was analyzed by lives, and by amounts; also according to sex and classes of insurance (life, endowment, etc.), all according to the duration of policies, thus producing select tables.

The nearest age at entry was taken, and the nearest duration method used. Duplicate lives were excluded whether of the same class or not, except for ascertaining the rate of claim, i. e., mortality by amounts. The treatment of withdrawals followed the British Offices' Experience, except that exact calculations were made for the first year.

An investigation of the manner in which the mortality changed during the period was made, by tracing the new business of different periods separately, and dividing 1829–1896 into four parts, namely: (1) 1829–1852, (2) 1852–1867, (3) 1867–1881, and (4) 1881–1896. The mortality of each period was compared with (1) the whole experience 1829–1896, and (2) the partial experience 1852–1896 by convenient age groupings for (*a*) the first five years, and (*b*) the sixth and subsequent years. The results showed that the mortality diminished from period to period, especially during the first five insurance years, indicating an improvement in methods of selection.

The final graduated Select Tables are joined to the ultimate table after 7 years. The first year's mortality declines from age 15 to age 28, and until age 40 continues less than for age 15; this characteristic is observed in a less pronounced form in subsequent durations.

The practice of the company had been to accept many risks on endowment plans where the excess mortality might be expected to fall mainly after maturity of the endowment; indeed this seems to be the only method used for treatment of substandard lives. Endowment policies were divided into three classes: (1) "Voluntaries," i. e., those who apply for such of their own accord, (2) "Involuntaries," i. e., those upon whom such policies were imposed, and (3) a class of policies issued as collateral for loans. These three divisions taken together show a mortality lighter than that for whole life policies, being 84% during the first five years and 96.3% thereafter. The voluntary endowment class shows an unusually low mortality and each of the other classes a mortality

considerably in excess of the whole life, the involuntary class being slightly the worst, the heavier mortality of this last named class being also distinctly observed during the first five years after entry.

The female experience was not extensive, there being 2,735 deaths. During the first year women appear as favorably as men, but during the second and subsequent years for ages under forty there is a noticeably higher mortality amongst females. From about age 40 onward the excess of male mortality over female is quite appreciable.

Japanese Life Tables.*

These tables are the first to give the mortality of an Oriental race, scientifically deduced from life insurance records. The tables, formed by Mr. K. Ebihara, F. I. A., were published in 1912. The material, furnished by three Japanese life companies, is of recent character, the oldest company having been founded in 1881. The experience closed in 1905 and to avoid special risks incurred during the Russo-Japanese war, all emigrants and lives in military service effecting policies in 1903 or after, were excluded.

The system of investigation followed closely that of the British Offices' Life Tables. 484,815 cards were sent in, of which 203,-143 related to endowments. Special features were:

1. Very heavy mortality at young ages and marked improvement between ages 20 and 30—generally heavier mortality than British or American standard.
2. The method of handling selection. The actual deaths in the first year were 62 per cent. of the expected by the Ultimate Japanese Table. This ratio of 62 per cent. was used for all ages; similarly for the second, third, and fourth years, the ratios were 87 per cent. 95 per cent. and 97 per cent.
3. The use of the function $B_{[x]+t}$, meaning "Brought down" or "net movement."

* J. I. A., Vol. XLVII., pp. 100–105.

MORTALITY OF ANNUITANTS.

GOVERNMENT ANNUITANTS—No. I, 1829.

In 1808 the National Debt Commissioners of Great Britain commenced to grant life annuities, and they applied to Mr. W. Morgan, the Actuary of the Old Equitable, to compute tables for this purpose; these tables he based on the Northampton Table. In 1819 Mr. John Finlaison who had been appointed Government Actuary pointed out that the prices charged for annuities were too small, and that the Government was losing heavily. Speculators had discovered the excellent values in such annuities, and had been buying annuities on carefully selected lives, sometimes also protecting themselves against loss by life insurance on the same lives. The loss to the government in the eleven years from 1808 amounted to nearly $10,000,000, yet a period of about ten more years elapsed before the error was corrected. Mr. Finlaison had in the meantime been taking steps to ascertain the rate of mortality amongst annuitants. He took for his materials the nominees of·

1. Various English and Irish Tontines from 1693 to 1789;
2. Life annuities issued at the Exchequer from 1745 to 1779, and
3. Life annuities chargeable on Sinking Fund from 1808 (as above).

In each case the facts were carefully tabulated, and in extracting them duplicate lives were eliminated. The ages last birthday were given in each case, and on an average the lives were assumed to be half a year older. Those stated to be 30 were assumed on the average to be $30\frac{1}{2}$, the sum of the numbers at 29 and 30 were assumed to give double the number exposed to risk at age 30.

Mr. Finlaison drew up twenty-one tables, two of which he adjusted by the formula:

$$\text{Graduated } p_x = \tfrac{1}{25}[p'_{x-4} + 2p'_{x-3} + 3p'_{x-2} + 4p'_{x-1} + 5p'_x + 4p'_{x+1} + 3p'_{x+2} + 2p'_{x+3} + p'_{x+4}].$$

called Finlaison's Method of Graduation.

There is reason to believe, however, that he subjected his data to some modification before applying his method of adjustment. One of the most important features of this investigation was the marked superiority of female life over male. Values of annuities were deduced from his results and employed by the government.

GOVERNMENT ANNUITANTS—No. II, 1860.

This table was formed by Mr. A. G. Finlaison (son of Mr. John Finlaison), the Actuary of the National Debt, the materials used being the nominees of:

1. The Irish Tontines of 1773–1777 ..... 3,384 persons.
2. The English Tontines of 1789 ..... 8,171 persons.
3. The annuities granted by National Debt Commissioners 1808–1850 ..... 16,812 persons.

28,367

Of these 28,367 persons, 11,829 were males and 16,538 females; 19,434 had died during the period over which the observations extended; while amongst the males were 675 carefully selected lives nominated by speculators who had purchased the annuities. This practice has been stopped by law. The Tables were never extensively used, and are now of but little interest.

GOVERNMENT ANNUITANTS—No. III, 1883.

Formed by Mr. A. J. Finlaison (grandson of Mr. John Finlaison), Actuary to the National Debt Commission, exclusively from the records of the annuities issued from the National Debt Office during the period from 1808 to 1875.

The number included was 30,788, of whom 10,929 were males, and 19,859 females. The 675 selected nominees previously referred to were also included amongst the males. 22,998 had died during the period and 7,790 were alive at the close of the observations. The tables published give the following particulars for each age:

1. Number of entrants.
2. Number alive at close of observations.
3. Deaths, and
4. Exposed to risk.

In the case of those alive at the close of the observations—1875—they were each kept under observation until their birthdays that year in order to avoid fractional years of age. The following is an extract from the table of elementary facts, age at purchase 60:

NUMBER OF ENTRANTS 475.

Age.	Alive at Close of Observations.	Deaths.	Number Exposed to Risk.
60	–	4	316.7
61	4	16	467
62	6	11	445
63	7	14	427
etc.	etc.	etc.	etc.

The ages at purchase of the annuities were stated as those last birthday, and it was found that, on an average, the annuitants were actually four months older. Accordingly, the true average age of the 475 entrants above mentioned was 60⅓; and up to the age of 61 these were exposed to risk for two-thirds of a year. Mr. Finlaison, to get the mortality for the first year, and for age 60 at entry, divided the four deaths above mentioned by two-thirds of 475, i. e., 316.7, this being equivalent to an assumption that the rate of mortality during the four months from the birthday to purchase was the same as for the eight months immediately following. A similar assumption was of course made for other ages. For the second year's mortality at age 61, the four who died, and the four alive at close of observations must be deducted from the entrants, thus giving 467 exposed to risk at age 61, among whom 16 deaths occurred, which showed a rate of mortality of .0343; similarly for subsequent years. The formula for deducing the exposed to risk would therefore be:

$$E_{[x]+n} = E_{[x]+n-1} - d_{[x]+n-1} - e_{[x]+n};$$

or

$$= E_{[x]} - \sum_1^n e - \sum_0^n d.$$

and for the general mortality table where the entrants at all ages were combined:

$$E_x = \Sigma n_{x-1} + \tfrac{2}{3} n_x - \Sigma e_x - \Sigma d_{x-1}$$

$$= E_{x-1} + \tfrac{1}{3} n_{x-1} + \tfrac{2}{3} n_x - e_x - d_{x-1},$$

where n_x represents the new entrants.

An analysis of the facts was made to ascertain the effect of the selection which the annuitants exercise, and it was found to be unimportant after four years. The entrants at each age were accordingly traced separately for the first four years of their existence (for ages 40 to 80) and thereafter combined with the entrants at all previous ages, thus giving Select Mortality Tables.

The treatment was equivalent to a "life-year" method, the annuitants being traced on the average from birthday to birthday. It has been suggested that the method of graduation by Woolhouse's formula distorted the table, and resulted in higher mortality rates than the facts justified; but this is now only of academic interest.

GOVERNMENT ANNUITANTS—No. IV, 1910.*

The Actuary to the National Debt Commissioners of Great Britain, Mr. J. Blakey, submitted a report dated 12th October, 1910, in which he gave particulars of an investigation into the mortality of government annuitants from 1st January, 1875, to 31st December, 1903, including new annuities issued during that time and annuitants living on the anniversary of purchase date in 1875. Where two or more annuities were granted, only the experience of the first was included. The following table gives a summary of the data:

GOVERNMENT ANNUITY EXPERIENCE, 1875–1904. SUMMARY OF DATA.

	Males.	Females.	Total.
Number of lives under observation	5,504	13,863	19,367
Number of years of risk	57,652	163,378	221,030
Number of deaths	4,168	9,333	13,501
Number of lives existing at the close of the observation	1,336	4,530	5,866

The investigation proceeds on the policy year plan, tracing the mortality from the date of admission to the next anniversary, and so forth, from anniversary to anniversary of the purchase of the annuity, following the method of the latest British Offices' Experience. The ages at entry were taken as the nearest age at date of purchase. An investigation as to the effect of using this showed that the assumption had the effect of understating the tabular ages by about 20 days in the case of male lives and about 15 days in the case of female lives—a trivial difference.

A considerable number of annuities are purchased at the National Debt Office under wills, etc., where the nominees do not exercise any option, but in most cases the nominees themselves are the purchasers.

The new tables are based on the assumption that the effects of selection are practically exhausted after the expiration of four years and the rates of mortality were, therefore merged into the aggregate table after that period. The tables were graduated by Makeham's method, satisfactorily in the case of aggregate tables, but modified for select tables in a manner similar to that employed in the graduation of the British Offices Annuitant

* J. I. A., XLVII, p. 66.

Tables. The following table compares the new values with other important tables of annuitant mortality.

ANNUITY VALUES—THREE PER CENT.

Age.	Males.			
	Government Annuitants, Select.		British Offices, Select.	McClintock's.
	1808–1875.	1875–1904.		
40	$16.37	$17.34	$17.60	$17.41
50	13.83	14.24	14.40	14.29
60	10.64	10.82	10.88	10.73
70	7.36	7.45	7.44	7.15
80	4.64	4.59	4.54	4.08
	Females.			
40	$18.17	$18.69	$18.26	$19.32
50	15.28	15.78	15.51	16.03
60	11.82	12.29	12.23	12.28
70	8.06	8.59	8.41	8.42
80	5.02	5.27	5.05	5.01

BRITISH OFFICES' LIFE ANNUITY TABLES, 1893.*

This investigation relates entirely to the experience of annuitants under contracts purchased from 43 life assurance and annuity institutions, including the British annuity experience of three American companies. Particulars of each annuity current at the commencement of the observations in 1863 and of those subsequently granted up to 1893 were supplied upon cards, a separate card being written for each annuity. The total of such cards was 9,700 for male and 24,300 for female lives, reduced by elimination of duplicates, etc., to 8,641 and 23,056, respectively. In the select male section 4,214 were existing; there had been 4,427 deaths; and 67,250 years of exposure. The corresponding figures in the female section were 11,956, 11,100, and 207,324. The tables are published in the form of select tables, the entrants at each age having been traced separately during the first five years, the rates of mortality thereafter being those of a modified ultimate experience formed by omitting the observations of the select period.

It had long been felt that the government tables were not applicable to the annuitants' experience of life assurance com-

* See "An Account of the Principles and Methods adopted in the Compilation of the Data, etc.," C. & E. Layton, London, 1903.

panies, the rates of mortality in the latter being lighter than the rates in the government tables. The fact that the observations for the government experience extended from 1808 to 1875 was in itself a deterrent influence, since it was believed that mortality had continued to improve. Moreover, the selection exercised against insurance companies is probably more intelligent, because government annuities are often bought for small amounts under the provisions of a will to pension old servants. Still further, Life Office annuitants are doubtless of a better social class than the government nominees. In the case of females, forming the great proportion of the annuitants, the British Offices annuity values approximate to those of the government at an age one year younger. In the case of the males at the older ages the difference is about half a year. The difference in reserves in case of a "model office" is roughly 3½%.

NOTATION.

Male Annuitants Table aggregate $O^{(m)}$ Female $O^{(f)}$
" " " select $O^{[m]}$ " $O^{[f]}$

The graduation was performed by Mr. George F. Hardy. A preliminary graduation of the Aggregate Table excluding the first five years, showed that the male mortality from 40 upward could be well represented by Makeham's formula. The female mortality could only be thus represented at ages above 65, and a supplementary curve had to be introduced below that age. The constants for these curves were obtained by equating to zero the sum of the deviations of the adjusted and unadjusted deaths, also the sum of the accumulated deviations.

Three trial graduations of the male table were made, for assumed values of log c equal to .040, .038, and .036; and the final value adopted was .038 as giving on the whole the most practical results, while following closely the ungraduated facts. An arbitrary adjustment was made in one of the constants, so as to make the male annuitants' mortality at the younger ages nearly the same as the mortality amongst assured lives. The facts at these ages were meagre; and, but for this adjustment the annuity values of the annuitants would have been appreciably smaller (i. e., higher mortality) than those of assured lives.

The graduation of the Select Tables for the first five years was made by introducing into Makeham's formula a function dependent upon duration after entry.

McClintock Annuity Tables.

In 1896 a paper appeared in the Transactions of the Actuarial Society of America* giving particulars of the annuity experience of fifteen American companies collected by Mr. Rufus W. Weeks, Actuary of the New York Life. At first all the annuities issued prior to 1890 were used and this material was afterwards carried forward to 1892 with additional new issues in the meantime. A feature of the experience was the preponderance of foreign business; about three fourths of the lives were European and one fourth American. The experience was based on lives; duplicates were eliminated. The age was taken at the birthday nearest to the date of first exposure; observation began on entry into any one of the companies and continued until death or the anniversary in 1892 when the experience closed. The exposure of a few deferred annuities in the experience was taken from the date at which the first annuity payment became due—at which date there is usually an option to take a cash settlement. Men and women were investigated separately, and complete select data is given for each age at entry. The number of lives included in the experience was 4,365 men, and 4,821 women.

Mr. McClintock took this data and derived two aggregate mortality tables† for male and female annuitants respectively. They were graduated by Makeham's Formula and have a common value for the constant c—namely: $\log c = .04$. Mr. McClintock stated that the tables differ somewhat from those which might be formed from a mere adjustment of Mr. Weeks' data but that he had endeavored to admit only such diversions as seem reasonable. His female table, for example, shows a somewhat higher mortality at ages under 70, because he considered that the experience at those ages had been materially affected by the large proportion of recent entries. With reference to the male table Mr. McClintock stated that he thought it best, if any departure from Mr. Weeks' experience should be permitted, that such departure should be on the safe side. That is to say, the mortality in Mr. McClintock's adjustment would be lighter than that actually shown by a direct application of the experience.

This table has been adopted by the State of New York and several other states as the standard for annuity valuations. The

* T. A. S. A., Vol., IV, .p. 275.

† T. A. S. A., Vol. VI., pp. 13 and 137.

following table illustrates the manner in which McClintock figures vary from those of other standard tables:

COMPLETE EXPECTATION OF LIFE $\mathring{e}_x$.

Males.				Females.			
Age.	Government 1883 Select.	Om Select.	McClintock Aggregate.	Age.	Government 1883 Select.	Of Select.	McClintock Aggregate.
40	26.04	28.31	28.08	40	29.84	30.16	32.47
50	20.26	21.23	21.11	50	22.92	23.49	24.53
60	14.38	14.82	14.64	60	16.26	17.01	17.22
70	9.33	9.54	9.18	70	10.30	10.88	11.00

ANNUITANTS RESIDENT IN THE UNITED STATES AND CANADA.

An unusually low rate of mortality amongst annuitants resident in the United States and Canada was indicated by the investigation into the experience of American and Canadian companies by Arthur Hunter in 1904. As the number of lives observed at that time was relatively small, the Council of the Actuarial Society recommended to the companies that additional data be furnished when the then available statistics should be doubled. This condition was fulfilled in 1910; and the companies supplied their data up to the year of issue 1909, the exposures being carried to the anniversaries in 1910. All the companies represented in the Actuarial Society granting annuities furnished their figures. Only immediate annuities were included, issued on single lives for the whole of life, under which the consideration had been paid in cash. Accordingly all temporary, deferred, joint, and survivorship annuities were excluded, as were also annuities issued in exchange for dividends.

In the experience to the anniversaries of the annuities in 1910 thirty companies contributed 12,174 cards, 6,620 relating to women, and 5,554 relating to men; more than three-quarters of the experience was supplied by six companies. After eliminating simultaneous contracts, there were 5,510 annuities on women, with 1,585 deaths; and 4,042 (42 per cent. of the total) on men with 1,295 deaths. The average ages at purchase were 63 for women and 61½ for men. The nearest integral age was taken, and the duration of the existing was obtained by deducting the year of purchase from 1910. Duplicates were brought together by sorting, (1) according to year of birth, and (2) according to name.

As compared with the British Offices' Annuity Tables (Select), the mortality among women was 84 per cent. by lives and 87 per cent. by amounts of annuity. While among men the corresponding percentages were 85 per cent. and 95 per cent. respectively, both sexes indicating a lower mortality in the United States and Canada than in Great Britain. There was no evidence that the mortality differed widely by ages at entry in its relation to the British Offices' Annuity Table.

A comparison of the experience to the anniversaries of 1904 with that to the anniversaries of 1910 showed that there had been very little change in the experience for the 6th and succeeding annuity years, the ratio of the 1910 table being for all ages and years of issue 85 per cent. of the British Select Annuity experience for men and 88 per cent. for women. During the first five annuity years, however, the experience on the issues of 1904 to 1910 showed a marked increase over the experience for the same annuity years to the anniversaries in 1904. Two hypotheses have been advanced as explaining this change: (*a*) that there has been greater care in obtaining proof of the ages of the annuitants at the date of purchase; and (*b*) that the proportion of people seeking annuities was lower than formerly, and the proportion of those actively canvassed was higher,—hence less selection by the annuitants.

Danish Survivorship Annuity Table.*

This table formed by Christian Jensen is based upon data derived from the experiences of voluntary survivorship annuities, granted by "Statsanstalten for Livsforsikring," the Danish State's Life Insurance Institution. The investigation deals with female beneficiaries—Danish women only. No tables were computed showing death rates of the insured lives. Obligatory annuities on Civil Servants were not included.

The period of observation is 1842 to 1900 and the experience is by lives, carried forward until the last policy for any particular beneficiary terminated. Exact ages from birthday to birthday were used, and the observations ceased on the birthday of the beneficiary in 1900, or with the first birthday after death prior to 1900. The graduation was by Makeham's Formula. The constant $\log c = .04579609$ is the same as that used in Hunter's Makehamized American experience, thus facilitating the use of

* T. A. S. A., Vol. X., p. 253. *Survivorship Annuity Tables.* Dawson, p. vi.

the two tables for joint lives. The other constants were derived by the "Method of Least Squares" from the table at age 40 upwards, involving 4,041 deaths.

The experience showed an improvement in the death rate towards the end of the period. The mortality of the Danish population is said to be as good as that in the healthier parts of the United States, so that the tables may be used until American Tables for Survivorship Annuities are available. In the absence of other authoritative tables showing the rates of mortality of annuitants under Survivorship Annuity Contracts, this table has been adopted as the standard for workmen's compensation allowances in the State of New York.

MISCELLANEOUS MORTALITY INVESTIGATIONS.

In addition to the tables herein described there have been many other investigations, some of which are of great interest; but space will not permit that they be discussed in detail, although references to the more important of these experiences may with advantage be given. Those who may have occasion to study any particular phase of mortality may thus have some knowledge of the lines of research and may be guided in the way towards which their further inquiries can profitably be directed.

These special investigations may be placed in general divisions as follows:

1. Investigation of death rates of different races;
2. Mortality rates in various countries and localities;
3. Observations according to class or occupation;
4. The experience of individual insurance companies.

The mortality of different races has not been scientifically analysed with any completeness. Of course, it is necessary that the various peoples should be under similar conditions in order that proper comparisons may be made. At various times investigations into the mortality of the colored race in the United States have been undertaken, as in the Specialized Investigation of the Actuarial Society and the U. S. Life Tables, 1910. The experience of individual companies on the same subject has also been dealt with, but the results have seldom been made public, although they have frequently been stated incidentally.

The mortality rates in different countries may be viewed either as affecting Caucasians resident in foreign lands, or as affecting the native population. Conditions as affecting whites resident in tropical countries have changed greatly in recent years through a better knowledge of malaria and other tropical diseases which are frequently transmitted by causes formerly unsuspected, often by mosquito bites. Yellow fever has disappeared from certain countries, while notable changes have been observed in such regions as Cuba and the Panama Canal Zone, making the older investigations of mortality in such regions of doubtful value. Some of the less progressive countries do not seem to have taken advantage of this growth of scientific knowledge.

The Journal of the Institute of Actuaries contains many refer-

ences to mortality in different countries, as for example mortality in certain parts of Africa;* the mortality in certain parts of Australia;† the mortality among the natives of India;‡ assured lives in the West Indies,§ etc. Much valuable information with reference to the mortality in semi-tropical and tropical countries is given in the Transactions of the Actuarial Society, Vol. X, p. 395; and by James Chatham in Trans. 3rd International Congress, p. 338.

Under the division of mortality according to class or occupation one most important publication is the Supplement to the Report of the Registrar General in England and Wales, Part II, published in 1908, dealing with the death rates in different occupations in England during the years 1900–'01–'02. This report succeeded another of the same nature, less complete, published ten years earlier. The question of mortality amongst dealers in alcoholic liquors was investigated by the Associated Scottish Life Offices,‖ also in the Specialized and Medico-Actuarial Investigations. The question of comparative mortality among abstainers and non-abstainers from the use of alcohol has been a subject of much controversy.** Those who use alcohol freely are unquestionably on the average poor risks for insurance purposes. The low mortality amongst the clergy generally was long ago indicated by investigations made by the late James Meikle into the Church of Scotland Ministers' Widows Fund, and this experience has been confirmed again and again, the most recent investigation of this type being that into the Presbyterian Ministers' Fund by L. G. Fouse, details of which have not been published.

Several interesting tables have also been prepared showing the mortality of British Peerage Families, the statistics being taken from books published from time to time giving information regarding the individual members of the Peerage. These publications also afforded a means of investigating the numbers of marriages and of births, and thus computing premiums for insurance against the birth of issue.†† Peerage and clergy statistics have also been used for determining the mortality in infancy and in childhood

* Vol. XXXIII, p. 285, XLVI, 308.
† Vol. XXXVI, p. 151.
‡ Vol. XXV, p. 217, XLIII, 365.
§ Vol. XXVII, p. 161.
‖ See J. I. A., Vol. XXXIII, p. 245.
** See paper and discussion, J. I. A., Vol. XXXVIII, p. 213; correspondence, same Vol., p. 273; and Transactions 5th International Congress, Vol. I, pp. 517–545.
†† See especially J. I. A., Vol. XXVIII, p. 350.

which has depended in the past very greatly upon the class of the parents, so that the ordinary census tables of mortality in infancy are not applicable for calculations relating to children's endowments.*

There have been many important investigations conducted by individual companies which have furnished information on particular points. For example, the Mutual Benefit Life Insurance Company investigated especially the mortality on policies continued under Extended Insurance.† Several companies, following the Mutual Life investigation of 1857, have taken up the causes of death of insured lives. These have not been discussed much by actuaries though they prove of great interest to medical men. The experience of the Connecticut Mutual Life Insurance Co. from its commencement to 1878 was published in 1884, and amongst other features showed that the mortality under term insurance had been excessive as compared with other classes, but it is explained that this might partly be accounted for by term policies taken out by the early Californian miners. Tables of discontinuance were also given, but under present day conditions these are not now of much value. More recent rates of discontinuance were given by the New York Life,‡ but even these rates have been greatly improved upon since 1907.

The information contained in the Medico-Actuarial publication with reference to height and weight was recently supplemented by the publication of "Standard Mortality Ratios incident to Variations in Height and Weight among men" compiled by a joint committee of the Actuarial Society and the Association of Medical Directors. This publication deals not only with deviations from standard weight as affected by age, but treats of (1) Medium sized, (2) Tall, and (3) Short men; also (4) Abdominal girth of stout men. A Practical Rating for Overweights was submitted by A. A. Welch in T. A. S. A., XVII, p. 17.

The Washington Life Insurance Company published mortality results in 1889 containing amongst other interesting data an investigation into the rates of mortality amongst policy holders taking their dividends as reversionary additions as compared with those taking cash. The latter were found to be very much the better lives confirming the results of a previous investigation by Mr .G. F. Hardy.§ The mortality experience of the Provident

* See J. I. A., Vol. XVII, p. 26.
† T. A. S. A., Vol. X, p. 597.
‡ T. A. S. A., Vol. IX, p. 103.
§ J. I. A., Vol. XXIII, p. 1.

Life & Trust Company from 1866 to 1875 showed unusually low mortality rates, apparently due in part to the large proportion of endowment policies issued by the company. Further investigations of the same company up to 1911 (T. A. S. A., XIV, p. 277) show low mortality in Endowment policies as compared with Life, and unexpectedly low rates in Term policies. An interesting investigation of mortality by plan of insurance, as experienced by the Aetna Life Insurance Company, appears in Vol. XVII, p. 246. The mortality rates applicable to policy issues from 1885 to 1905 are relatively high in relation to the entire experience; while all issues indicate a low mortality during the years from 1905 to 1913.

The experience on deferred dividend policies after the dividend period expires, policies being continued, was submitted by Arthur Hunter, Vol. XIV, p. 38. The results show a clear condition of selection by policyholders adverse to the company. On more than one occasion it has been shown that when an automatic provision for extended insurance or even for paid-up insurance goes into effect, there is a tendency towards high mortality in the first year or two after lapse. Probably the policyholders "allowed their policies to lapse as a direct result of serious impairment in health and without regard to consequence."*

These and many other phases of the mortality question have been ably discussed, yet the conditions of life have been changing so rapidly that results obtained some years ago while possibly indicating the trend of events or showing that certain causes may lead to good or evil effects, are not necessarily applicable to present day conditions. Accordingly there is almost unlimited scope for further research, and we may expect to see a still closer scientific study leading especially towards the improvement of conditions in unhealthy regions or occupations.

*T. A S. A., XV, p. 303.

TABLE SHOWING THE RATE OF MORTALITY (q_x) ACCORDING TO CERTAIN TABLES.

Age.	American Experience.	Actuaries Table.	Thirty American Offices. (Males).	H^M	O^M	$O^{M(5)}$	Age.
20	.0078	.0073	.0068	.0063	.0040	.0065	20
25	.0081	.0078	.0070	.0066	.0048	.0069	25
30	.0084	.0084	.0075	.0077	.0060	.0075	30
35	.0090	.0093	.0082	.0088	.0074	.0084	35
40	.0098	.0104	.0094	.0103	.0092	.0098	40
45	.0112	.0122	.0112	.0122	.0115	.0120	45
50	.0138	.0159	.0142	.0160	.0150	.0154	50
55	.0186	.0217	.0189	.0210	.0204	.0208	55
60	.0267	.0303	.0265	.0297	.0289	.0292	60
65	.0401	.0441	.0386	.0434	.0420	.0422	65
70	.0620	.0649	.0578	.0622	.0621	.0622	70
75	.0944	.0956	.0878	.0984	.0926	.0927	75
80	.1445	.1404	.1341	.1447	.1384	.1385	80

Age.	Gotha 1852–95 Whole Life —Males.	Healthy English No. 1 (Males).	English Life No. 3 (Males).	English Life No. 6 (Males)	Carlisle Table.	North-ampton Table.	Age.
20	.0053	.0070	.0083	.0046	.0071	.0140	20
25	.0047	.0078	.0092	.0057	.0073	.0158	25
30	.0045	.0082	.0101	.0067	.0101	.0171	30
35	.0058	.0086	.0113	.0090	.0103	.0187	35
40	.0078	.0094	.0130	.0119	.0130	.0209	40
45	.0104	.0108	.0154	.0148	.0148	.0240	45
50	.0144	.0130	.0188	.0194	.0134	.0284	50
55	.0213	.0166	.0246	.0257	.0179	.0335	55
60	.0320	.0237	.0325	.0360	.0335	.0402	60
65	.0467	.0368	.0459	.0497	.0411	.0490	65
70	.0709	.0556	.0673	.0721	.0516	.0649	70
75	.1061	.0840	.0988	.1054	.0955	.0962	75
80	.1619	.1249	.1418	.1520	.1217	.1343	80

TABLE SHOWING THE COMPLETE EXPECTATION OF LIFE ($\mathring{e}_x$) ACCORDING TO CERTAIN TABLES.

Age.	American Experience.	Actuaries' Table.	Thirty American Offices. (Males.)	H^M	$O^{[M]}$	O^M	$O^{M(5)}$	Age.
20	42.2	41.5	43.1	42.1	42.9	43.7	42.4	20
25	38.8	38.0	39.5	38.4	39.2	39.6	38.7	25
30	35.3	34.4	35.8	34.7	35.6	35.6	35.1	30
35	31.8	30.9	32.2	31.0	31.9	31.7	31.4	35
40	28.2	27.3	28.5	27.4	28.2	27.9	27.7	40
45	24.5	23.7	24.8	23.8	24.7	24.2	24.0	45
50	20.9	20.2	21.2	20.3	21.2	20.6	20.5	50
55	17.4	16.9	17.8	17.0	17.9	17.2	17.2	55
60	14.1	13.8	14.6	13.8	14.9	14.1	14.0	60
65	11.1	11.0	11.6	11.0	12.1	11.2	11.2	65
70	8.5	8.5	9.0	8.5	9.7	8.7	8.7	70
75	6.3	6.5	6.7	6.4	7.6	6.6	6.6	75
80	4.4	4.8	4.9	4.7		4.8	4.8	80

Age.	Gotha		Healthy English No. 1 (Males).	English Life No. 3 (Males).	English Life No. 6 (Males).	Carlisle Table.	North-ampton Table.	Age.
	Select.	Aggregate.						
20	43.0	43.6	43.4	39.5	41.0	41.5	33.4	20
25	39.2	39.7	39.9	36.1	37.0	37.9	30.8	25
30	35.3	35.6	36.4	32.8	33.1	34.3	28.3	30
35	31.3	31.4	32.9	29.4	29.2	31.0	25.7	35
40	27.4	27.4	29.3	26.1	25.6	27.6	23.1	40
45	23.6	23.5	25.6	22.8	22.2	24.5	20.5	45
50	20.0	19.8	22.0	19.5	18.9	21.1	18.0	50
55	16.8	16.3	18.5	16.4	15.8	17.6	15.6	55
60	13.9	13.1	15.1	13.5	12.9	14.3	13.2	60
65	11.4	10.4	12.0	10.8	10.3	11.8	10.9	65
70		7.9	9.4	8.4	8.0	9.2	8.6	70
75		5.9	7.2	6.5	6.2	7.0	6.5	75
80		4.3	5.4	4.9	4.6	5.5	4.8	80